

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>09 APR 2015</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-2015 to 00-00-2015</b>	
4. TITLE AND SUBTITLE <b>Evaluation of the Utility of Static and Adaptive Mesh Refinement for Idealized Tropical Cyclone Problems in a Spectral Element Shallow Water Model</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Naval Research Laboratory, Marine Meteorology Division, Monterey, CA, 93943</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT <b>The utility of static and adaptive mesh refinement (SMR and AMR, respectively) are examined for idealized tropical cyclone (TC) simulations in a spectral element f-plane shallow water model. The SMR simulations have varying sizes of the statically refined meshes (geometry-based) while the AMR simulations use a potential vorticity (PV) threshold to adaptively refine the mesh to the evolving TC. Numerical simulations are conducted for four cases: (i) TC-like vortex advecting in a uniform flow, (ii) binary vortex interaction, (iii) barotropic instability of a PV ring, and (iv) barotropic instability of a thin strip of PV. For each case, a high resolution ???truth??? simulation is compared to two different SMR simulations and three different AMR simulations for accuracy and efficiency. The multiple SMR and AMR simulations have variations in the number of fully-refined elements in the vicinity of the TC. For these idealized cases, it is found that the SMR and AMR simulations are able to resolve the vortex dynamics as well as the ???truth??? runs, with no significant loss in accuracy in the refined region in the vortex vicinity and with significant speed-ups (factor of 2-5). The overall accuracy is enhanced by a greater area of fully refined mesh in both the SMR and AMR simulations. While these results are highly idealized, they demonstrate the potential for SMR and AMR for the numerical simulation of TCs in three dimensions and more complex models.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>54</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			



1 **Evaluation of the utility of static and adaptive mesh refinement**  
2 **for idealized tropical cyclone problems in a spectral element**  
3 **shallow water model**

4 **ERIC A. HENDRICKS \***

*Marine Meteorology Division, Naval Research Laboratory, Monterey, CA, USA*

**MICHAL A. KOPERA AND FRANCIS X. GIRALDO**

*Department of Mathematics, Naval Postgraduate School, Monterey, CA, USA*

**MELINDA S. PENG, JAMES D. DOYLE, AND QINGFANG JIANG**

*Marine Meteorology Division, Naval Research Laboratory, Monterey, CA, USA*

*Mon. Wea. Rev*

Submitted: April 9, 2015

---

\*Corresponding author address: Eric A. Hendricks, Naval Research Laboratory, Monterey, CA 93943

E-mail: eric.hendricks@nrlmry.navy.mil

## ABSTRACT

The utility of static and adaptive mesh refinement (SMR and AMR, respectively) are examined for idealized tropical cyclone (TC) simulations in a spectral element  $f$ -plane shallow water model. The SMR simulations have varying sizes of the statically refined meshes (geometry-based) while the AMR simulations use a potential vorticity (PV) threshold to adaptively refine the mesh to the evolving TC. Numerical simulations are conducted for four cases: (i) TC-like vortex advecting in a uniform flow, (ii) binary vortex interaction, (iii) barotropic instability of a PV ring, and (iv) barotropic instability of a thin strip of PV. For each case, a high resolution “truth” simulation is compared to two different SMR simulations and three different AMR simulations for accuracy and efficiency. The multiple SMR and AMR simulations have variations in the number of fully-refined elements in the vicinity of the TC. For these idealized cases, it is found that the SMR and AMR simulations are able to resolve the vortex dynamics as well as the “truth” runs, with no significant loss in accuracy in the refined region in the vortex vicinity and with significant speed-ups (factor of 2-5). The overall accuracy is enhanced by a greater area of fully refined mesh in both the SMR and AMR simulations. While these results are highly idealized, they demonstrate the potential for SMR and AMR for the numerical simulation of TCs in three dimensions and more complex models.

## 1. Introduction

Atmospheric motions span a multitude of different spatial and temporal scales. Examples are planetary waves at spatial scales of  $10^6$  m which evolve over days to boundary layer

turbulent eddies at scales of  $10^1$  m which evolves over minutes. With current computational resources, it is not possible to simulate the entire spectrum of atmospheric flows. One of the goals in the design of next generation numerical weather prediction (NWP) models is that they be unified, or that one nonhydrostatic dynamical core has the capability of simulating a wide-spectrum atmospheric spatial and temporal scales of motion, from microscale to global, and weather to climate. Severe and high-impact weather can often take the form of localized weather systems, such as severe thunderstorms, tornadoes, fronts, and tropical cyclones. With limited computational resources, it would be ideal to perform local mesh refinement to resolve the details of these features, while resolving the large-scale features (e.g., synoptic scale high pressure systems) at coarser resolution.

Currently, the primary method for tackling this scale discrepancy is by utilizing multiply nested numerical weather prediction (NWP) models (Kurihara et al. 1979; Hodur 1997; Kurihara et al. 1998; Skamarock et al. 2005; Doyle et al. 2014). However, a number of drawbacks exist with this method. First, there exist multiple lateral boundaries, often with the existence of non-physical blending zones. Secondly, there is inefficiency in performing the same forecast on each nest since the nests are embedded within each other. Thirdly, due to the extra communication required between nests, it is not expected that these setups would scale that efficiently on multiple processors. An alternative method to embedded nests is static or adaptive mesh refinement. In the former method, a mesh could be statically refined over a region of interest (e.g., a city or coastline) providing more fine scale details of the flow there. In the latter method, the mesh could adaptively refine and de-refine based on some feature of interest, such as a tropical cyclone (TC). The earlier work of Berger and Oliger (1984) and Skamarock and Klemp (1993) demonstrated the utility of AMR for hyperbolic

equations. A review of the current state of AMR for atmospheric modeling is described in Jablonowski (2004) and Behrens (2006).

The purpose of the present study is to examine the utility of both SMR and AMR for ideal TC simulations in a next generation dynamical core. The model is a planar spectral element shallow water model, with similar numerical methods used in the Nonhydrostatic Unified Model of the Atmosphere (NUMA; Giraldo and Restelli 2008). We examine the utility of SMR and AMR for four flows, representing idealizations of TC dynamics in the real atmosphere. First, we examine the a TC advecting in a uniform flow, representing a TC tracking in the atmosphere in steady environmental flow. Secondly, a binary vortex interaction is examined, representing the interaction of two TCs that are close together. Thirdly, instabilities and mixing processes are examined in the hurricane inner-core (eye and eyewall). Fourth, the instability of the intertropical convergence zone (ITCZ), its breakdown, and formation of TC-like vortices. In each case, we compare a series of SMR and AMR simulations with variable regions of refined mesh to a “truth” simulation with uniform refined mesh in order to obtain an understanding of efficiency and accuracy tradeoffs. The remainder of this paper is organized as follows. In section 2, the continuous model equations and numerical method are given. Each experimental setup is given in section 3, along with details of the spatial and temporal discretization. The results from each experiment are given and discussed in section 4. A summary of the main findings are given in section 5.

## 2. Model equations and numerical method

### a. Continuous equations

The model is based upon the divergent barotropic (shallow water) equations in Cartesian coordinates on an  $f$ -plane. The governing equations are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v + g \frac{\partial h}{\partial x} = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + f u + g \frac{\partial h}{\partial y} = 0 \quad (2)$$

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} + \frac{\partial(vh)}{\partial y} = 0, \quad (3)$$

where  $u$  is the zonal momentum per unit mass,  $v$  is the meridional momentum per unit mass,  $h$  is the fluid depth, and  $f$  is the Coriolis parameter. An important property of the unforced, inviscid shallow water equations (1)-(3) is the material conservation of potential vorticity,

$$\frac{DP}{Dt} = 0, \quad (4)$$

where  $P = (f + \zeta)/h$  is the potential vorticity, where  $\zeta = \partial v / \partial x - \partial u / \partial y$  is the relative vorticity and  $D/Dt = (\partial/\partial t) + u(\partial/\partial x) + v(\partial/\partial y)$  is the material derivative.

### b. Numerical method and mesh refinement algorithms

The flux form of the continuous shallow water equations (1)–(3) are discretized using the continuous Galerkin (CG), or spectral element, numerical method. The flux form shallow water equations are written in compact vector form as follows:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \left( \frac{\mathbf{U} \otimes \mathbf{U}}{\varphi} + \frac{1}{2} \varphi^2 \mathbf{I}_2 \right) + f \mathbf{k} \times \mathbf{U} = 0 \quad (5)$$

$$\frac{\partial \varphi}{\partial t} + \nabla \cdot \mathbf{U} = 0, \quad (6)$$

where  $\mathbf{U} = \varphi \mathbf{u}$ ,  $\mathbf{u} = (u, v, 0)^T$  is the velocity vector (where the superscript  $T$  denotes the transpose),  $\varphi = gh$  with  $h$  being the fluid height and  $g$  the gravitational constant. Other quantities requiring definition include:  $\mathbf{I}_2 \in \mathcal{R}^2$  is an identity matrix,  $\mathbf{k} = (0, 0, 1)^T$  is the vector pointing upwards (along the z-coordinate which coincides with the direction along which  $h$  is measured),  $\otimes$  denotes the tensor product operator, and  $\nabla \cdot$  denotes the divergence operator.

High order CG methods for the shallow water equations are given in Ma (1993) and Taylor et al. (1997), and the numerical model used in this study is based upon the specific methods discussed in Giraldo (2001) and Giraldo and Restelli (2008). The spectral element method has been applied to numerous idealized test cases, as well as more complicated idealized tests cases of atmospheric phenomena, such as moist experiments of a squall line (Gabersek et al. 2012).

A brief overview of the elemental CG method is given here. Given a computational domain  $\Omega$ , the domain is first decomposed into a number of elements  $N_e$  as

$$\Omega = \bigcup_{e=1}^{N_e} \Omega_e \quad (7)$$

where  $\Omega_e$  is one element. In each element, the weak integral form of the shallow water equations above is taken, and the solution is expanded as

$$q_N(x, y, t) = \sum_{j=1}^{M_N} \chi_j(x, y) q_j(t) \quad (8)$$

where  $q_N$  is a prognostic variable,  $M_N = (N + 1)^2$ ,  $N$  is the polynomial order, and  $\chi_j$  is a



local basis function. In the CG method, neighboring elements share interface points and in each element the solution is obtained at the Legendre-Gauss-Lobatto (LGL) nodal points. As an example, Fig. 1 shows the LGL nodal points inside one element for  $N = 5$ .

The mesh refinement algorithm is based upon Kopera and Giraldo (2014, 2015), and uses a forest of quad trees (i.e., each internal node has four children), similar to the approach used by St-Cyr et al. (2008). In the refinement procedure, the polynomial order  $N$  is held constant, while the mesh is refined (i.e.,  $h$ -refinement is done instead of  $p$ -refinement). For the experiments here, a maximum of two levels of mesh refinement is used, so that a fully refined element is four times the horizontal resolution of a coarse element. Additionally, the refinement algorithm includes the functionality to generate an arbitrary number of layers of refined mesh cells extending away from the feature of interest. This is hereafter referred to as the “buffer” region. The AMR criterion for this study is the potential vorticity  $P$ , and refinement and coarsening of the elements is accomplished based on a threshold in  $P$ . An attractive feature of PV is that linear inertia-gravity waves have zero PV, eliminating the possibility of AMR tracking fast-mode inertia-gravity waves. The SMR criterion is based subjectively on different area sizes around the feature of interest.

### 3. Initial conditions and model setup

Four different test cases are examined and described in detail below. These cases are: (i) TC vortex moving in a uniform flow, (ii) binary vortex interaction, (iii) dynamic instability of the hurricane eyewall, eye mesovortex formation and mixing between the eyewall and eye, and (iv) formation of TC-like vortices from the barotropic instability of a shear zone. These

111 cases are idealizations of TC dynamics occurring in the real atmosphere. In order to relate  
 112 the simulations to the real processes, Fig. 2 shows an example real-case scenario that each test  
 113 case is designed to represent. In Fig. 2a, Hurricane Andrew is shown moving west towards  
 114 Florida, being advected by the easterly flow around the subtropical ridge to its north (case  
 115 1 idealization). In Fig. 2b, the binary vortex interaction of two storms Typhoons Melor and  
 116 Parma are shown. The differential advection induced from each cyclone advects the other  
 117 creating a net cyclonic motion (the Fujiwhara effect; Fujiwhara 1921) (case 2 idealization).  
 118 In Fig. 2c, the instability and break down of the eyewall of Hurricane Dolly (2008) is shown,  
 119 leading to an asymmetric radar reflectivity pattern there (case 3 idealization). Finally, in  
 120 Fig. 2d, the instability and breakdown of the ITCZ is shown over days. The deep convection  
 121 along the ITCZ is observed to undulate and finally breakdown into distinct tropical cyclones  
 122 (case 4 idealization).

123 *a. Case 1: TC vortex advecting in a uniform flow*

The first test case is a TC-like vortex advecting in a uniform flow, which is an idealiza-  
 tion of a TC moving with the environmental flow in the atmosphere. The initial vortex is  
 constructed as a Rankine vortex in polar coordinates  $(r, \phi$ , where  $r = (x^2 + y^2)^{1/2}$  and  $\phi$  is  
 the azimuthal angle in radians) according to

$$v_\phi(r, \phi, 0) = \begin{cases} \zeta_1 r/2 & 0 \leq r \leq r_1, \\ \zeta_1 r_1^2/(2r) & r_1 \leq r < \infty, \end{cases} \quad (9)$$

where  $v_\phi$  is the tangential velocity,  $\zeta_1 = 1 \times 10^{-3} \text{ s}^{-1}$  and  $r_1 = 50 \text{ km}$ . With these parameters,  
 the peak tangential velocity at  $r = 50 \text{ km}$  is  $25 \text{ m s}^{-1}$ . A smooth radial decay function

$(1 - r/r_{\text{cut}})^2$  is added to the tangential winds so that  $v_\phi(r, \phi, 0) = 0$  at  $r_{\text{cut}}$ , with a cutoff radius  $r_{\text{cut}} = 220$  km. The vortex Cartesian momentum components  $u$  and  $v$  are next specified and then the uniform zonal flow  $u_0 = 10 \text{ m s}^{-1}$  is added to  $u$ . This experiment is done on an  $f$ -plane with  $f = 0$ . The initial balanced fluid depth is determined by solving the nonlinear balance equation

$$g\nabla^2 h = f\nabla^2 \psi - 2 \left[ \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 - \frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} \right], \quad (10)$$

124 using the CG method, where  $\zeta = \nabla^2 \psi$ ,  $u = \partial \psi / \partial y$  and  $v = -\partial \psi / \partial x$ .

125 The equations are solved on a square domain of  $600 \text{ km} \times 600 \text{ km}$ . The setup is a  
126 zonal channel flow, with no-flux boundary conditions applied at the north and south lateral  
127 boundaries, and periodic boundary conditions applied at the west and east boundaries. The  
128 simulation is run for one revolution, so the final TC is located at the starting point, for  
129 comparison to the analytic solution which is the initial condition.

#### 130 *b. Case 2: Binary vortex interaction*

131 The second case is a binary vortex interaction (Dritschel and Waugh 1992; Prieto et al.  
132 2001, 2003). In this case, two TC-like vortices are offset by a certain distance and allowed  
133 to interact with one another. Depending on the offset distance, and the size and intensity  
134 of each vortex, different interactions can occur such as complete merger, complete straining  
135 out, partial straining out, elastic interaction (Prieto et al. 2003). The case we choose here  
136 is an elastic interaction, where each vortex retains its shape, but the interaction of the two  
137 vortices cause a net cyclonic motion (the Fujiwhara effect).

138 The initial condition for the binary vortex interaction case consists of two offset Rankine

139 vortices. Each vortex is constructed according to

$$v_\phi(r, \phi, 0) = \begin{cases} \zeta_1 r/2 & 0 \leq r \leq r_1, \\ \zeta_1 r_1^2/(2r) & r_1 \leq r < \infty, \end{cases} \quad (11)$$

140 where  $\zeta_1 = 1 \times 10^{-3} \text{ s}^{-1}$ . The first vortex is positioned at  $(x, y) = (15, 0)$  km and the second  
 141 vortex is positioned at  $(x, y) = (-15, 0)$  km. The same  $r_{\text{cut}}$  as in case 1 is applied to each  
 142 vortex to ensure the winds decay to zero before the lateral boundary. No-flux boundary  
 143 conditions are used at each lateral boundary. The nonlinear balance equation (8) is solved  
 144 using the initial condition of both vortices in order to obtain the corresponding  $h(x, y, 0)$   
 145 field, only with  $f = 1.0 \times 10^{-4} \text{ s}^{-1}$ .

### 146 *c. Case 3: Barotropic instability of the hurricane eyewall*

147 The third case is the barotropic instability of the hurricane eyewall. The hurricane eyewall  
 148 can largely be described as a three-region model: (i) low vorticity eye, (ii) high vorticity  
 149 eyewall, and (iii) low vorticity environment. Observations of such vorticity structures in real  
 150 hurricanes are given in Kossin and Eastin (2001) and Hendricks et al. (2012). An idealization  
 151 of this structure can be constructed according to Schubert et al. (1999) using the tangential  
 152 velocity profile

$$v_\phi(r, \phi, 0) = \frac{1}{2r} \begin{cases} \xi_1 r^2 + \xi_2 r^2 & 0 \leq r \leq r_1 \\ \xi_1 r_1^2 + \xi_2 r^2 & r_1 \leq r \leq r_2, \\ \xi_1 r_1^2 + \xi_2 r_2^2 & r_2 \leq r \leq \infty \end{cases} \quad (12)$$

which defines a discrete three region model of axisymmetric relative vorticity

$$\zeta(r, \phi, 0) = \frac{1}{r} \frac{\partial(rv)}{\partial r} = \begin{cases} \xi_1 + \xi_2 & 0 \leq r \leq r_1 \\ \xi_2 & r_1 \leq r \leq r_2, \\ 0 & r_2 \leq r \leq \infty \end{cases} \quad (13)$$

Here  $\xi_1 = -3 \times 10^{-3} \text{ s}^{-1}$ ,  $\xi_2 = 3 \times 10^{-3} \text{ s}^{-1}$ ,  $r_1 = 40 \text{ km}$  and  $r_2 = 50 \text{ km}$ . The hurricane eye is defined as the region less than  $r_1$ , the eyewall is defined as the region between  $r_1$  and  $r_2$ , and the environment is defined as the region between  $r_2$  and infinity. Similar to the previous experiments, a smooth radial decay function  $(1 - r/r_{\text{cut}})^2$  is added to the tangential winds so that  $v_\phi(r, \phi, 0) = 0$  at  $r_{\text{cut}}$ . The cutoff radius  $r_{\text{cut}} = 220 \text{ km}$ . No-flux boundary conditions are used at each lateral boundary.

The nature of the instability is as follows. Each vorticity gradient of the ring supports a vortex Rossby wave (Montgomery and Kallenbach 1997). The inner vortex Rossby wave progrades relative the mean flow, while the outer vortex Rossby wave retrogrades relative to the mean flow. Thus it is possible for each of these waves to have the same angular velocity, or be phase-locked, leading to the barotropic instability of the ring. A comprehensive linear stability analysis of this structure is provided by Schubert et al. (1999), and nonlinear simulations and discussions of aspects of this problem are given in Kossin and Schubert (2001) and Hendricks et al. (2009).

To initiate the instability process, a broadband perturbation was added to the basic state

vorticity (13) of the form

$$\zeta'(r, \phi, 0) = \zeta_{amp} \sum_{m=1}^8 \cos(m\phi + \phi_m) \times \begin{cases} 0 & 0 \leq r \leq r_1, \\ 1 & r_1 \leq r \leq r_2, \\ 0 & r_2 \leq r < \infty, \end{cases} \quad (14)$$

where  $\zeta_{amp} = 1.0 \times 10^{-5} \text{ s}^{-1}$  is the amplitude and  $\phi_m$  the phase of azimuthal wavenumber  $m$ . For this set of experiments, the phase angles  $\phi_m$  were chosen to be random numbers in the range  $0 \leq \phi_m \leq 2\pi$ . In real hurricanes, the impulse is expected to develop from a wide spectrum of background turbulent and convective motions. Similar to case 1, the nonlinear balance equation (8) is solved to obtain the corresponding  $h(x, y, 0)$  field, with  $f = 1.0 \times 10^{-4} \text{ s}^{-1}$ . This ring has a thickness parameter  $\delta = r_1/r_2 = 0.8$  and hollowness parameter  $\gamma = (\xi_1 + \xi_2)/\zeta_{av} = 0$  (where  $\zeta_{av}$  is the average inner-core vorticity). According to the linear stability analysis of Schubert et al. (1999), this ring is most unstable to azimuthal wavenumber  $m = 5$ , with an  $e$ -folding time of 0.57 h.

#### *d. Case 4: Barotropic instability of a shear zone*

The fourth test is the examination of the formation of TC-like vortices through the barotropic instability of a region of large horizontal velocity shear. An example of such a process occurring in the real atmosphere is the instability of the inter-tropical convergence zone (ITCZ) in the eastern North Pacific ocean basin, causing it to undulate over days, and eventually break down and form pools of high PV (Fig. 2d). Provided favorable conditions, these PV pools can then form into TCs. An observational and modeling study of the genesis of TCs from this formation mechanism is given in Ferreira and Schubert (1997).

184 A similar experiment is conducted here. The initial condition is constructed as an ideal-  
 185 ization of the ITCZ, with easterly flow to the north and westerly flow to the south. A linear  
 186 function is assumed to bridge the two regions, forming a thin strip of cyclonic vorticity.  
 187 Mathematically, the initial condition is

$$u(x, y, 0) = \begin{cases} -u_0 & y \geq y_0, \\ -u_0 y / y_0 & -y_0 \leq y \leq y_0, \\ u_0 & y \leq -y_0, \end{cases} \quad (15)$$

where  $u_0 = 20 \text{ m s}^{-1}$  and  $y_0 = 100 \text{ km}$ . Here,  $v(x, y, 0) = 0$  and  $h$  is determined by solving the geostrophic balance equation analytically,

$$h(x, y, 0) = \begin{cases} f u_0 y / g & y \geq y_0, \\ [f u_0 / (2g y_0)](y^2 + y_0^2) & -y_0 \leq y \leq y_0, \\ -f u_0 y / g & y \leq -y_0, \end{cases} \quad (16)$$

188 where  $f = 1.0 \times 10^{-4} \text{ s}^{-1}$ .

189 The strip of PV supports the existence of two counter-propagating Rossby waves, one  
 190 on the northern PV gradient which propagates to the east and one on the southern PV  
 191 gradient which propagates to the west. These two waves can phase-lock and grow, leading  
 192 to the break down of the strip into vortices. A linear stability analysis of this structure is  
 193 given by Gill (1980), and here we provide a basic overview of the stability characteristics.  
 194 Assuming separation of the meridional and zonal structure, Rossby wave solutions of the  
 195 form  $\psi'(x, y, t) = \Psi(y) \exp(ik(x - ct))$  are sought, where  $\psi'$  is the wave streamfunction.  
 196 Here,  $c = c_r + ic_i$  is the complex phase velocity, and  $k$  is the zonal wavenumber. The linear

stability analysis of this simple shear zone indicates the most unstable zonal wavenumber  $k = 0.3984/y_0 = 3.984 \times 10^{-6} \text{ m}^{-1}$ , or approximately 1577 km. The domain used here is  $8000 \text{ km} \times 8000 \text{ km}$ , therefore the most unstable mode is zonal wavenumber-5. The growth rate  $kc_i = 0.2012u_0/y_0 = 4.024 \times 10^{-5} \text{ s}^{-1}$ , corresponding to an  $e$ -folding time of 6.9 hours. In order to initiate the instability, a weak amplitude zonal wavenumber-5 perturbation in vorticity is applied to the region of constant background vorticity (shear zone). The lateral boundary conditions for this run are the same as case 1, no-flux conditions are applied at the north and south boundaries and periodic conditions are applied at the west and east boundaries.

#### *e. Discretization and model setup*

For all simulations, 5th order polynomials ( $N = 5$ ) are used in each element, and a fourth order explicit Runge-Kutta scheme is used for the temporal integration. No diffusion is used in the experiments, however a modal filter is applied to help control nonlinear instability. For each initial condition listed above, six numerical simulations are performed: (i) a high resolution “truth” simulation (FINE), (ii) A large statically refined mesh around the TC processes (SMR2), (iii) a smaller statically refined mesh (SMR1), (iv) adaptive mesh refinement with a buffer of 6 fully refined elements (AMR3), (v) adaptive mesh refinement with a buffer of 3 fully refined elements (AMR2), and (vi) adaptive mesh refinement with no buffer (AMR1). The FINE numerical simulation is the “truth”, and is expected to simulate the phenonema with the most accuracy. The varying SMR and AMR simulations are intended to help retain the accuracy of the solution in the vicinity of the TC while also saving on



218 computational time. All simulations are run at the time step of the FINE simulation since  
 219 our goal is understanding computational aspects only with regard to the spatial variation.  
 220 The discretization, horizontal resolution, and model setup parameters for all experiments  
 221 are given in Table 1. Since  $N = 5$  polynomials are used in each element, an approximate  
 222 effective resolution can be obtained by dividing the element size by a factor of 5. However, the  
 223 actual minimum grid spacing is less than this number since the LGL points are unequally  
 224 spaced, and closer together near the element boundary (Fig. 1).

## 225 4. Results

226 In this section, the qualitative results of each simulation are described, followed by a  
 227 quantitative analysis of the solution accuracy and computational aspects. For cases 1 and 2  
 228 which simulate vortex advection, the results are described in terms of the prognostic variables  
 229 of zonal and meridional velocity. For cases 3 and 4 which simulate barotropic instability,  
 230 the results are described in terms of the PV in order to better illustrate the salient dynamics.

### 231 a. Case 1

232 The initial condition of case 1 is given in Fig. 3. Here the magnitude of the perturbation  
 233 velocity vector  $((u - u_0)^2 + v^2)^{1/2}$  for each simulation is shown in colored contours, with the  
 234 elements overlayed. Note that only the element boundaries are overlayed, and not the actual  
 235 grid of nodal points inside each element. The uniform resolution mesh (FINE simulation)  
 236 is shown in Fig. 3a. In Fig. 3b, the SMR2 simulation is shown which has fully refined mesh

237 for  $-200 \text{ km} < y < 200 \text{ km}$ . The SMR1 initial condition is shown in Fig. 3c, which has fully  
 238 refined mesh for  $-100 \text{ km} < y < 100 \text{ km}$ . The initial conditions for the AMR3, AMR2, and  
 239 AMR1 simulations are shown in Fig. 1d,e,f, respectively. Here initial condition is adapting  
 240 to the PV threshold providing fully-refined mesh around the hurricane eyewall (yellow region  
 241 of stronger winds) and eye. The AMR3, AMR2, and AMR1 buffers are readily evident as  
 242 refined mesh extending from the center.

243 In Fig. 4, the simulation results are shown after a half-revolution. At this time, each  
 244 vortex has moved a distance of 300 km to the right, and the mid-point of each vortex is  
 245 at the left and right lateral boundaries. The vortex core is well resolved in each simulation  
 246 and there are no apparent phase errors as the vortex center of each simulation is exactly at  
 247 the lateral boundary. The outer wind field also appears well resolved by the SMR2, SMR1,  
 248 AMR3, and AMR2 simulations. There exists some azimuthal variability in the outer wind  
 249 field in the AMR1 simulation. This could be from some stronger gravity wave activity due to  
 250 imbalances generated as a result of the coarser representation of the outer wind field. Finally,  
 251 moving to Fig. 5, each simulation is shown at the final time after one complete revolution,  
 252 so that the vortex is at its initial position. At this time, all simulations appear to resolve  
 253 the vortex core well, and there are again no apparent numerical advection errors (moving  
 254 too slow or fast). Again, here the AMR1 simulation has the most noticable differences in  
 255 the outer wind field. Overall, qualitatively the results show that all AMR/SMR simulations  
 256 are able to resolve the core of the TC as it advects in the uniform flow.

*b. Case 2*

The initial condition for the binary vortex interaction is given in Fig. 6. The two Rankine vortices are evident as the two wind maximas. All simulations are able to resolve the vortex core well. Here the SMR2 simulation has statically refined mesh between  $-200 \text{ km} < (x,y) < 200 \text{ km}$ , and SMR1 simulation has statically refined mesh between  $-100 \text{ km} < (x,y) < 100 \text{ km}$ . Moving to Fig. 7, each vortex is shown at  $t = 12 \text{ h}$ . The vortices have rotated in cyclonic motion approximately 135 degrees and the outer winds of each vortex has advected the other. At this time each simulation has a similar orientation of the binary vortices, indicating that even the AMR1 simulation is rotating the vortices at the correct angular velocity. In Fig. 8, the solution is shown at  $t = 24 \text{ h}$ , after another net cyclonic rotation of 135 degrees. At this time, all vortices appear to have a similar orientation with the exception of the AMR1 simulation, which has not rotated cyclonically in the proper amount due to the slight weakening of the vortex winds. However, all simulations do a reasonable job at capturing the vortex core wind velocity magnitude.

*c. Case 3*

The initial condition of case 3 is given in Fig. 9. Here, a vortex with a very sharp gradient in tangential velocity is shown (ring of elevated PV). The initial condition of each simulation has fully-refined mesh over the ring of elevated PV. The AMR1 simulation (with no buffer) has a couple of coarse mesh cells at the very center as the initial mesh is adapting to the ring of large PV only. In Fig. 10, the simulation is shown at  $t = 2.5 \text{ h}$ , as the most unstable mode of hybrid azimuthal wavenumbers  $m = 5/6$  is occurring in each simulation. In Fig. 11,

the simulation is shown at  $t = 5$  h, after the vortex has broken down into mesovortices (evident by separate PV anomalies). Each simulation appears to be resolving these localized features well. In Fig. 12, the simulation is shown as the mesovortices become strained and filamented, and begin to merge into one central monopole. At this time, the structure of the merging vortices looks quite similar in each run, demonstrating that the static and adaptive mesh refinement in the local region is working properly. Finally, in Fig. 13, the simulations are shown at  $t = 48$  h, after the initial PV ring has mixed into a monopole. All simulations qualitatively have similar structures. The AMR1 simulation has a slightly different orientation of the central monopole.

#### *d. Case 4*

The initial condition for case 4 is given for each simulation in Fig. 14. Decreasing amounts of refined mesh are evident in moving from the FINE to the AMR1 case, with the AMR1 case only have refined mesh over the PV strip itself. In Fig. 15, each simulation is shown at  $t = 45$  h. Each simulation produces the theoretically predicted most unstable mode of wavenumber-5. Upshear tilt of each PV anomaly associated with the northern and southern counter-propagating Rossby waves is evident. The simulations are shown at  $t = 180$  h in Fig. 16. Here the breakdown of the PV strip has resulted in the five separate vortices. All simulations are able to reproduce the five vortices.

## 296 *e. Error Norms*

297 In order to quantify aspects of these results, normalized L2 errors were computed between  
 298 the final state in the FINE, or “truth”, simulation, and the final state in the SMR/AMR  
 299 simulations. In order to compute the L2 error norms, the solution at the AMR/SMR meshes  
 300 at the final time are adapted to the FINE mesh. The normalized  $L_2$  error is defined as

$$L2 = \left( \frac{\sum_{k=1}^N (q^{\text{num}}(x_k) - q^{\text{ref}}(x_k))^2}{\sum_{k=1}^N q^{\text{ref}}(x_k)^2} \right)^{1/2} \quad (17)$$

301 where  $q$  is the predicted variable,  $N$  is the total number of points, the superscript “num”  
 302 denotes the numerical simulations, and the superscript “ref” denotes the reference solution  
 303 (or in this case the FINE solution). The normalized L2 error is computed for the magnitude  
 304 of the velocity vector  $|\mathbf{U} = (u, v)|$  and the geopotential  $\varphi = gh$ . Since we are interested in  
 305 how well the SMR/AMR simulations resolve the local TC processes in comparison to the  
 306 FINE simulations, the L2 errors are computed in two regions: (i) the entire domain, and  
 307 (ii) in the localized region in which the AMR/SMR simulations are designed to resolve. For  
 308 (ii), this region was defined as  $r < 100$  km for cases 1, 2, and 3, and  $|y| < 500$  km for case  
 309 4. These correspond to the regions of large PV in each case.

310 The L2 error norms for (i) and (ii) are shown in Figs. 17 and 18, respectively. In both  
 311 figures, the green bars are the L2 errors in the magnitude of the velocity vector and the black  
 312 bars are the L2 errors in the magnitude of fluid depth. In Fig. 17, for each case, there is a  
 313 general trend of decreasing L2 error norms moving from the AMR to the SMR to the FINE  
 314 simulations. In Fig. 17a, in terms of the velocity vector magnitude, the AMR1 simulation has  
 315 one order of magnitude larger error than the AMR2, AMR3, and SMR1 simulations ( $10^{-2}$

316 versus  $10^{-3}$ ). The SMR2 simulation has the lowest errors ( $10^{-4}$ ). A similar trend is evident  
 317 for  $\varphi$ , however the AMR1 simulation has a significantly larger error than the other runs. In  
 318 Fig. 17b, the results are shown for case 2. A similar trend is evident, however in this case the  
 319 AMR1 simulation L2 error in  $\varphi$  is not as significant. In Fig. 17c, a similar result is also seen in  
 320 the instability case. Moving to Fig. 17d, the results are broadly consistent with the previous  
 321 panels, however in this case the L2 error in  $\varphi$  shows a continuing decrease, rather than  
 322 asymptoting as in the previous results. Overall, the results for the entire domain indicate  
 323 that the SMR2 simulations are most accurate. There are not significant differences between  
 324 the AMR2, AMR3, and SMR1 simulations. In general, the AMR1 simulation typically has  
 325 larger errors than the other simulations. In Fig. 18, the same L2 error norms are shown  
 326 in the region of high PV. Broadly, the results are consistent with Fig. 17, however the  
 327 errors are generally lower. This is expected since the localized regions have only fully-refined  
 328 elements. In summary, these results indicate that very high accuracy may be obtained for  
 329 these TC simulations by using a large statically refined mesh (SMR2). However, the AMR  
 330 simulations with only three buffer elements (AMR2) are able to produce similar accuracy  
 331 of the 6 element buffer simulation (AMR3) and statically refined mesh (SMR1) simulations.  
 332 This is important, since as we will show in the next section, the computational expense of  
 333 AMR2 is significantly less than AMR3 and SMR1.

#### 334 *f. Computational aspects*

335 Each simulation was executed on a single central processing unit (CPU). In Fig. 19, some  
 336 computational aspects of the simulations are given. The point ratio is shown in Fig. 19a,

and in Fig. 19b, the speedup is given. The point ratio is defined as the inverse of the total number of points of the FINE simulation divided by the average number of points of the AMR simulations (since the points change in time), and the total fixed number of points in the SMR simulations. The speedup is defined as the CPU time of the FINE simulation divided by the CPU time of each of the other simulations. In Fig. 19a, for cases 1–3, the AMR1 simulation has approximately 9–12 times fewer nodal points than the FINE simulation. The AMR1 simulation of case 4 has approximately 5 times fewer points due to the different structure of this atmospheric phenomenon (zonal strip instead of a central vortex). There is approximately a linearly decreasing trend of the point ratio moving to the AMR2, AMR3, SMR1, and SMR2 simulations. Fig. 19b shows the speedup for each simulation over the FINE run. The AMR1 simulation has the largest speedup (factor of 3.5–5), there is a linearly decreasing trend of speedup moving to the FINE simulation. Overall, the speedup is approximately one-half of the point ratio. Since the time step of each simulation is identical, if there were no overhead in refining and de-refining elements, one would expect the speedup factor to be similar to the point ratio. However, this overhead leads to lesser speedups.

#### *g. TC vortex moving through a variable mesh*

It has been discussed that in next generation NWP models without AMR, a useful domain structure for simulation of TCs would consist of a large region of refined mesh over the entire tropics, with coarser mesh away from the tropics. In this scenario, while often the TC would remain in the tropics, re-curving TCs would move from the fully-refined mesh to the coarser mesh. It is important to understand how a TC may change in structure moving through

358 such an abrupt mesh boundary. One would expect that without any forcing, the maximum  
 359 wind speed in the eyewall would be reduced by moving from finely resolved mesh to a coarser  
 360 mesh, as the eyewall region is less well resolved. The results of this test are given in Fig. 20.  
 361 Here, the TC vortex is initially centered on a 200 km square box of fully-refined mesh, and  
 362 then advected to the right in uniform zonal flow (as in case 1). At  $t = 4.165$  h, half the  
 363 eyewall is in the coarse mesh, while half is in the fully-refined mesh. As expected, a slight  
 364 reduction in the tangential velocity is evident. As the vortex advects into the coarser mesh,  
 365 then back into the fine mesh, it loses kinetic energy. The fraction of final integrated kinetic  
 366 energy to initial integrated kinetic energy within  $r < 100$  km is 0.999939, indicating that  
 367 the loss is quite small. These results indicate that the high order methods used here can  
 368 even broadly preserve aspects of the vortex inner-core structure while moving through an  
 369 abrupt mesh boundary when the elements are quadrupled in size. This result is broadly  
 370 consistent with Zarzycki et al. (2014), who found little numerical distortion when a dry TC  
 371 vortex moved through an abrupt transition of a variable mesh using the spectral element  
 372 dynamical core of the Community Atmosphere Model (CAM-SE). More energy loss would  
 373 be expected if a lower polynomial order ( $N < 5$ ) or larger elements were used.

## 374 5. Conclusions

375 A planar shallow water model based on the continuous Galerkin (spectral element) nu-  
 376 merical method has been used to examine idealized tropical cyclone (TC) problems, with a  
 377 focus on the applicability of static and adaptive mesh refinement (SMR and AMR, respec-  
 378 tively). Four different idealizations of TC cases in the real atmosphere were simulated in this



379 model, with varying degrees of SMR and AMR. The SMR/AMR simulations were compared  
 380 to a high resolution “truth” simulation (noted previously as the FINE run) with regard to  
 381 solution accuracy and computational time. Three different AMR simulations were conducted  
 382 with varying levels of buffer regions (or the number of extra layers of fine elements added to  
 383 the finely resolved region). Two different SMR simulations were executed with varying levels  
 384 of refined mesh. For AMR simulations, a potential vorticity threshold was used for refining  
 385 and de-refining elements. With regard to solution accuracy, the SMR2 simulation (with the  
 386 largest area of fully refined mesh) was shown to be superior to the other simulations (at least  
 387 an order of magnitude lower L2 error) in comparison to the “truth” run. However, the AMR  
 388 simulations with only 3 buffer elements (AMR2) are shown to be as accurate overall as the  
 389 AMR simulations with 6 buffer elements (AMR3) and the smaller statically refined mesh  
 390 simulation SMR1. The AMR simulation with no buffer elements (AMR1) was generally  
 391 shown to be significantly less accurate than the others. Significant speed-ups were obtained  
 392 by using AMR. The AMR2 simulations (which are nearly as accurate as the AMR3 and  
 393 SMR1 simulations) had speed-ups of 2.5–4.5 over the FINE simulation. Thus, these results  
 394 indicate that AMR can be used at significantly less computational expense to resolve the  
 395 TC feature as well as the “truth” run, provided a sufficient buffer region exists.

396 In summary, we wish to note that we have examined static and adaptive mesh refinement  
 397 for TC applications in a very idealized framework of a shallow water fluid in constant rota-  
 398 tion. In the real atmosphere, TCs are three-dimensional phenomena, with complex physics  
 399 interactions (microphysics, boundary layer, vertical mixing, and radiation), as well as inter-  
 400 actions with the environment (such as vertical wind shear and ocean surface fluxes). One  
 401 of the major challenges in the future with AMR is the development of scale-aware physical

parameterizations that will seamlessly represent physical processes across scales. However, these results demonstrate that from a purely dry dynamical modeling standpoint, AMR shows great promise for TC applications.

#### *Acknowledgments.*

We gratefully acknowledge support from the Chief of Naval Research PE-0601153N. The codes used in this paper were developed by MAK and FXG during the 4-month long program on Multiscale Numerics in the Atmosphere and Ocean at the Newton Institute, Cambridge University. MAK and FXG gratefully acknowledge the support of the Office of Naval Research through program element PE-0602435N, the National Science Foundation (Division of Mathematical Sciences) through program element 121670, and the Air Force Office of Scientific Research through the Computational Mathematics program. We thank Sasa Gabersek, Alex Reinecke and Kevin Viner for helpful comments.

## REFERENCES

- 416 Behrens, J., 2006: Adaptive atmospheric modeling: key techniques in grid generation, data  
417 structures, and numerical operations with applications. Springer, 2006.
- 418 Berger, M. J. and J. Oliger, 1984: Adaptive mesh refinement for hyperbolic partial differential  
419 equations. *J. Comput. Phys.*, **53**, 484–512.
- 420 Doyle, J. D., R. Hodur, S. Chen, Y. J. and J.R. Moskaitis, S. Wang, E. A. Hendricks, H. Jin,  
421 and T. Smith, 2014: Tropical cyclone prediction using COAMPS-TC. *The Oceanography*  
422 *Soc.*, **27**, 104–115.
- 423 Dritschel, D. D. and D. W. Waugh, 1992: Quantification of the inelastic interaction of  
424 unequal vortices in two-dimensional vortex dynamics. *Phys. Fluids*, **4**, 1737–1744.
- 425 Ferreira, R. N. and W. H. Schubert, 1997: Barotropic aspects of ITCZ breakdown. *J. Atmos.*  
426 *Sci.*, **54**, 261–285.
- 427 Fujiwhara, S., 1921: The natural tendency towards symmetry of motion and its application  
428 as a principle in meteorology. *Quart. J. Roy. Meteor. Soc.*, **47**, 287–293.
- 429 Gabersek, S., F. X. Giraldo, and J. D. Doyle, 2012: Dry and moist idealized experiments  
430 with a two-dimensional spectral element model. *Mon. Wea. Rev.*, **140**, 3163–3182.
- 431 Gill, A. E., 1980: Some simple solutions for heat-induced tropical circulation. *Quart. J. Roy.*  
432 *Meteor. Soc.*, **106**, 447–462.

433 Giraldo, F. X., 2001: A spectral element shallow water model on spherical geodesic grids.  
434 *Int. J. Numer. Methods Fluids*, **35**, 869–901.

435 Giraldo, F. X. and M. Restelli, 2008: A study of spectral element and discontinuous Galerkin  
436 methods for the Navier–Stokes equations in nonhydrostatic mesoscale atmospheric mod-  
437 eling: Equation sets and test cases. *J. Comput. Phys.*, **227**, 3849–3877.

438 Hendricks, E. A., B. D. McNoldy, and W. H. Schubert, 2012: Observed inner-core structural  
439 variability in Hurricane Dolly (2008). *Mon. Wea. Rev.*, **140**, 4066–4077.

440 Hendricks, E. A., W. H. Schubert, R. K. Taft, H. Wang, and J. P. Kossin, 2009: Lifecycles  
441 of hurricane-like vorticity rings. *J. Atmos. Sci.*, **66**, 705–722.

442 Hodur, R. M., 1997: The Naval Research Laboratory’s Coupled Ocean/Atmosphere  
443 Mesoscale Prediction System (COAMPS). *Mon. Wea. Rev.*, **125**, 1414–1430.

444 Jablonowski, C., 2004: Adaptive grids in weather and climate modeling. Ph.D. Thesis, The  
445 University of Michigan (2004).

446 Kopera, M. A. and F. X. Giraldo, 2014: Analysis of adaptive mesh refinement for IMEX  
447 discontinuous Galerkin solutions of the compressible euler equations with application to  
448 atmospheric simulations. *J. Comput. Phys.*, **275**, 92–117.

449 Kopera, M. A. and F. X. Giraldo, 2015: Mass conservation of the unified continuous and  
450 discontinuous element-based Galerkin methods on dynamically adaptive grids with appli-  
451 cation to atmospheric simulations. *J. Comput. Phys.*, in press.

452 Kossin, J. P. and M. D. Eastin, 2001: Two distinct regimes in the kinematic and thermody-  
453 namic structure of the hurricane eye and eyewall. *J. Atmos. Sci.*, **58**, 1079–1090.

454 Kossin, J. P. and W. H. Schubert, 2001: Mesovortices, polygonal flow patterns, and rapid  
455 pressure falls in hurricane-like vortices. *J. Atmos. Sci.*, **58**, 2196–2209.

456 Kurihara, Y. M., G. J. Tripoli, and M. A. Bender, 1979: Design of a movable nested-mesh  
457 primitive equation model. *Mon. Wea. Rev.*, **107**, 239–249.

458 Kurihara, Y. M., R. E. Tuleya, and M. A. Bender, 1998: The GFDL Hurricane Prediction  
459 System and its performance in the 1995 hurricane season. *Mon. Wea. Rev.*, **126**, 1306–  
460 1322.

461 Ma, H., 1993: A spectral element basin model for the shallow water equations. *J. Comput.*  
462 *Phys.*, **109**, 133–149.

463 Montgomery, M. T. and R. J. Kallenbach, 1997: A theory for vortex Rossby waves and its  
464 application to spiral bands and intensity changes in hurricanes. *Quart. J. Roy. Meteor.*  
465 *Soc.*, **123**, 435–465.

466 Prieto, R., J. P. Kossin, and W. H. Schubert, 2001: Symmetrization of lopsided vorticity  
467 monopoles and offset hurricane eyes. *Quart. J. Roy. Meteor. Soc.*, **127**, 1–17.

468 Prieto, R., B. D. McNoldy, S. R. Fulton, and W. H. Schubert, 2003: A classification of  
469 binary tropical cyclone-like vortex interactions. *Mon. Wea. Rev.*, **131**, 2656–2666.

470 Schubert, W. H., M. T. Montgomery, R. K. Taft, T. A. Guinn, S. R. Fulton, J. P. Kossin,

and J. P. Edwards, 1999: Polygonal eyewalls, asymmetric eye contraction, and potential vorticity mixing in hurricanes. *J. Atmos. Sci.*, **56**, 1197–1223.

Skamarock, W. C. and J. B. Klemp, 1993: Adaptive grid refinement for two-dimensional and three-dimensional nonhydrostatic atmospheric flow. *Mon. Wea. Rev.*, **121**, 788–804.

Skamarock, W. C., J. B. Klemp, J. Dudhia, D. O. Gall, D. M. Barker, W. Wang, and J. G. Powers, 2005: A description of the Advanced Research WRF Version 2. Tech. rep., NCAR Tech. Note NCAR/TN-468+STR, 20 pp., Boulder, CO. [Available online at [http://www.mmm.ucar.edu/wrf/users/docs/arw\\_v2.pdf](http://www.mmm.ucar.edu/wrf/users/docs/arw_v2.pdf)].

St-Cyr, A., C. Jablonowski, J. M. Dennis, H. M. Tufo, and S. J. Thomas, 2008: A comparison of two shallow-water models with nonconforming adaptive grids. *Mon. Wea. Rev.*, **136**, 1898–1922.

Taylor, M., J. Tribbia, and M. Iskandarani, 1997: The spectral element method for the shallow water equations on the sphere. *J. Comput. Phys.*, **130**, 92–108.

Zarzycki, C., C. Jablonowski, and M. Taylor, 2014: Using variable-resolution meshes to model tropical cyclones in Community Atmosphere Model. *Mon. Wea. Rev.*, **142**, 1221–1239.

## 487 **List of Tables**

488	1	Experiment Parameters	29
-----	---	-----------------------	----

TABLE 1. Experiment Parameters

	<b>Case 1</b>	<b>Case 2</b>	<b>Case 3</b>	<b>Case 4</b>
Domain size (km)	$600 \times 600$	$600 \times 600$	$600 \times 600$	$8000 \times 8000$
Fully Refined				
Number of Elements	$60 \times 60$	$60 \times 60$	$60 \times 60$	$60 \times 60$
Element spacing (km)	10	10	10	133.33
Effective resolution (km)	2	2	2	26.2
Fully Unrefined				
Number of Elements	$15 \times 15$	$15 \times 15$	$15 \times 15$	$15 \times 15$
Element spacing (km)	40	40	40	533.33
Effective resolution (km)	8	8	8	106.67
Polynomial order	5	5	5	5
Model time step (s)	3	3	3	18



## List of Figures

- 1     Grid of Legendre-Gauss-Lobatto nodal points inside one element using  $N = 5$   
order polynomials as basis functions. 33
- 2     Satellite and radar observations depicting real TC processes that are being  
simulated in the idealized framework. Processes are highlighted in bold red.  
Panels: a) NOAA-12 visible satellite imagery of Hurricane Andrew advecting  
to the west toward Florida at 1231Z on 23 Aug 1992 (courtesy NOAA), b)  
MODIS visible satellite imagery of the binary vortex interaction of Tropical  
Storm Parma and Typhoon Melor on 6 Oct 2009 (courtesy NASA/GSFC), c)  
Radar image of Hurricane Dolly (2008) at 1002Z 7 Jul 2008 approaching the  
Texas coast (courtesy NOAA/NWS/KRBO). Significant azimuthal variability  
in the radar reflectivity in the eyewall is evident, and d) instability and break-  
down of the ITCZ into multiple TCs over the timescale of days (reproduced  
from Ferreira and Schubert (1997)). 34
- 3     Initial condition for case 1: advecting vortex. Panels: a) FINE, b) SMR2, c)  
SMR1, d) AMR3, e) AMR2, f) AMR1. 35
- 4     Simulations of case 1 after one-half revolution, at  $t = 8.33$  h. Panels: a)  
FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1. 36
- 5     Simulations of case 1 after a full revolution, at  $t = 16.67$  h. Panels: a) FINE,  
b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1. 37
- 6     Initial condition for case 2: Binary vortex interaction. Panels: a) FINE, b)  
SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1. 38

511	7	Simulations of case 2 at $t = 12$ h. Panels: a) FINE, b) SMR2, c) SMR1, d)	
512		AMR3, e) AMR2, f) AMR1.	39
513	8	Simulations of case 2 at $t = 24$ h. Panels: a) FINE, b) SMR2, c) SMR1, d)	
514		AMR3, e) AMR2, f) AMR1.	40
515	9	Initial condition for case 3: unstable vortex. Panels: a) FINE, b) SMR2, c)	
516		SMR1, d) AMR3, e) AMR2, f) AMR1.	41
517	10	Simulations of case 3 at $t = 2.5$ h as barotropic instability occurs. Panels: a)	
518		FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.	42
519	11	Simulations of case 3 at $t = 5$ h as mesovortices form. Panels: a) FINE, b)	
520		SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.	43
521	12	Simulations of case 3 at $t = 15.75$ h as the mesovortices merge into a monopole.	
522		Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.	44
523	13	Simulations of case 3 at $t = 48$ h as the PV ring has broken down and mixed	
524		to a monopole. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2,	
525		f) AMR1.	45
526	14	Initial condition for case 4: strip of PV representing the ITCZ. Panels: a)	
527		FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.	46
528	15	Simulations of case 4 as barotropic instability sets in at $t = 45$ h. Panels: a)	
529		FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.	47
530	16	Simulations of case 4 after multiple TC-like vortices have formed, at $t = 180$	
531		h. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.	48

532	17	Normalized L2 Errors for all cases for using all nodal points in the domain.	
533		Panels: a) ccase 1: advection, b) case 2: binary vortex, c) case 3: instability,	
534		d) case 4: ITCZ. The green bars represent the magnitude of the velocity vector	
535		and the black bars represent the geopotential.	49
536	18	Normalized L2 Errors for all cases using only nodal points in the localized	
537		region of high PV ( $r < 100$ km for cases 1–3, $ y  < 500$ km for case 4. Panels:	
538		a) case 1: advection, b) case 2: binary vortex, c) case 3: instability, d) case	
539		4: ITCZ. The green bars represent the magnitude of the velocity vector and	
540		the black bars represent the geopotential.	50
541	19	Panels: a) Ratio of the number of FINE nodal points to the average number	
542		of nodal points in each of the other simulations, and b) Ratio of the CPU	
543		time of the FINE simulation to the CPU time of each of the other simulations.	51
544	20	TC vortex moving through a fixed variable mesh. Panels: a) $t = 0$ h, b)	
545		$t = 4.165$ h, c) $t = 8.333$ h, d) $t = 12.5$ h, e) $t = 16.667$ h.	52

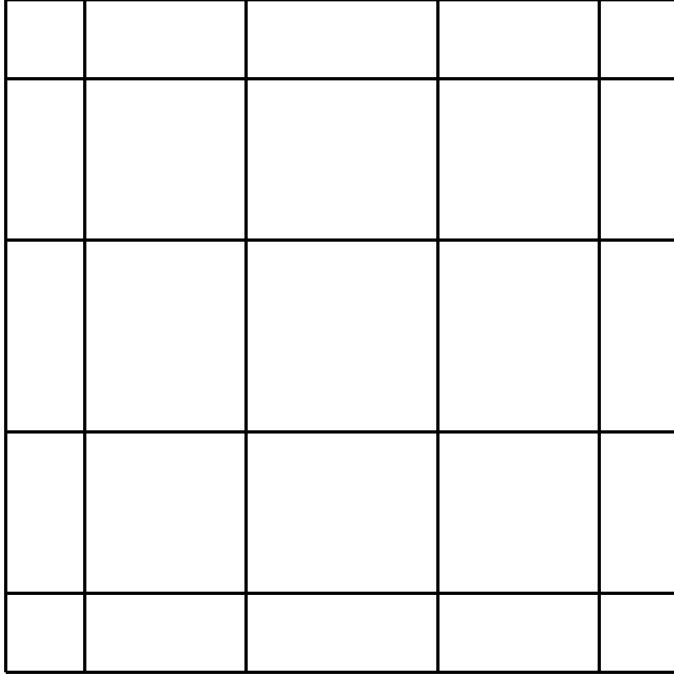


FIG. 1. Grid of Legendre-Gauss-Lobatto nodal points inside one element using  $N = 5$  order polynomials as basis functions.

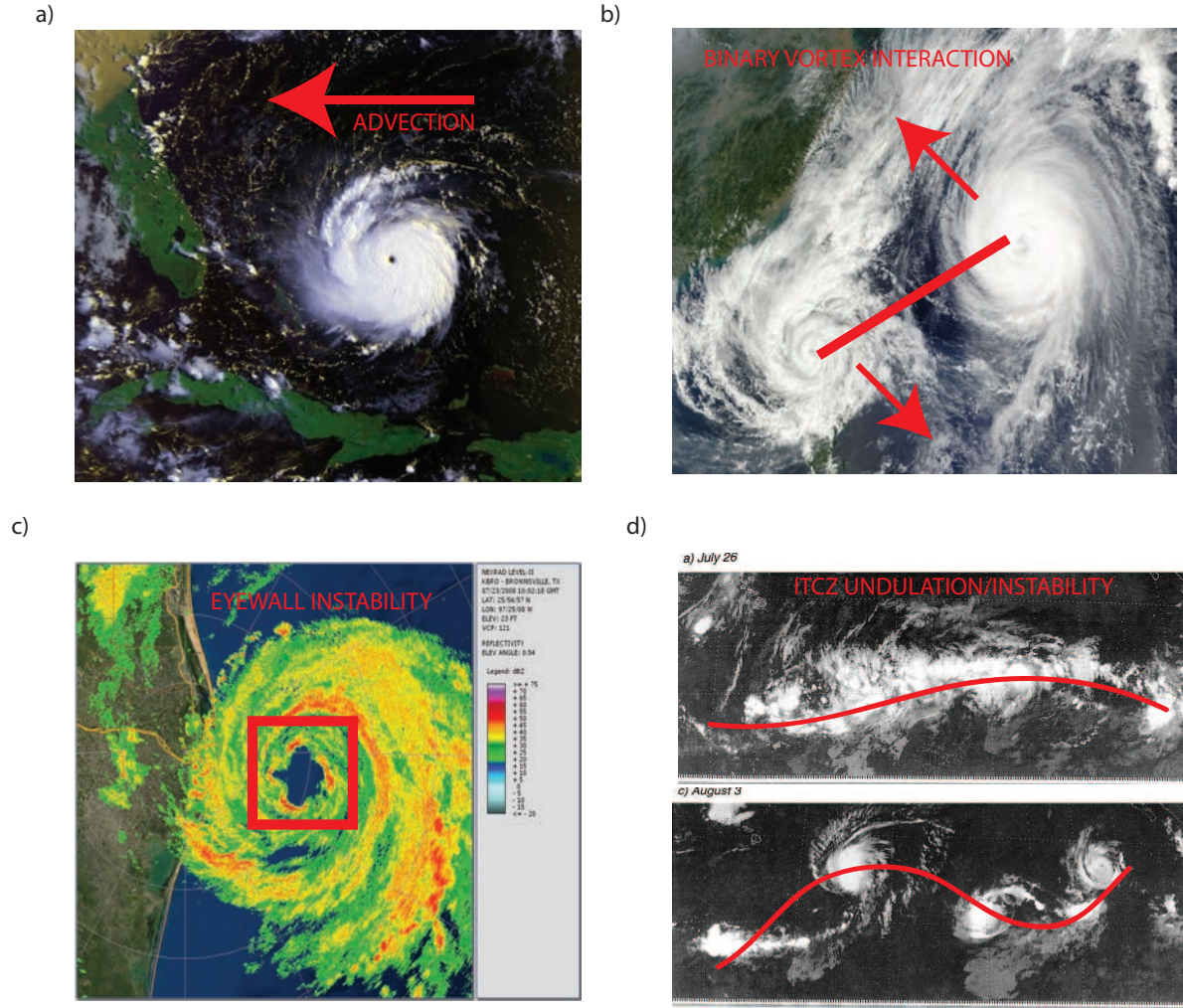


FIG. 2. Satellite and radar observations depicting real TC processes that are being simulated in the idealized framework. Processes are highlighted in bold red. Panels: a) NOAA-12 visible satellite imagery of Hurricane Andrew advecting to the west toward Florida at 1231Z on 23 Aug 1992 (courtesy NOAA), b) MODIS visible satellite imagery of the binary vortex interaction of Tropical Storm Parma and Typhoon Melor on 6 Oct 2009 (courtesy NASA/GSFC), c) Radar image of Hurricane Dolly (2008) at 1002Z 7 Jul 2008 approaching the Texas coast (courtesy NOAA/NWS/KRBO). Significant azimuthal variability in the radar reflectivity in the eyewall is evident, and d) instability and breakdown of the ITCZ into multiple TCs over the timescale of days (reproduced from Ferreira and Schubert (1997)).

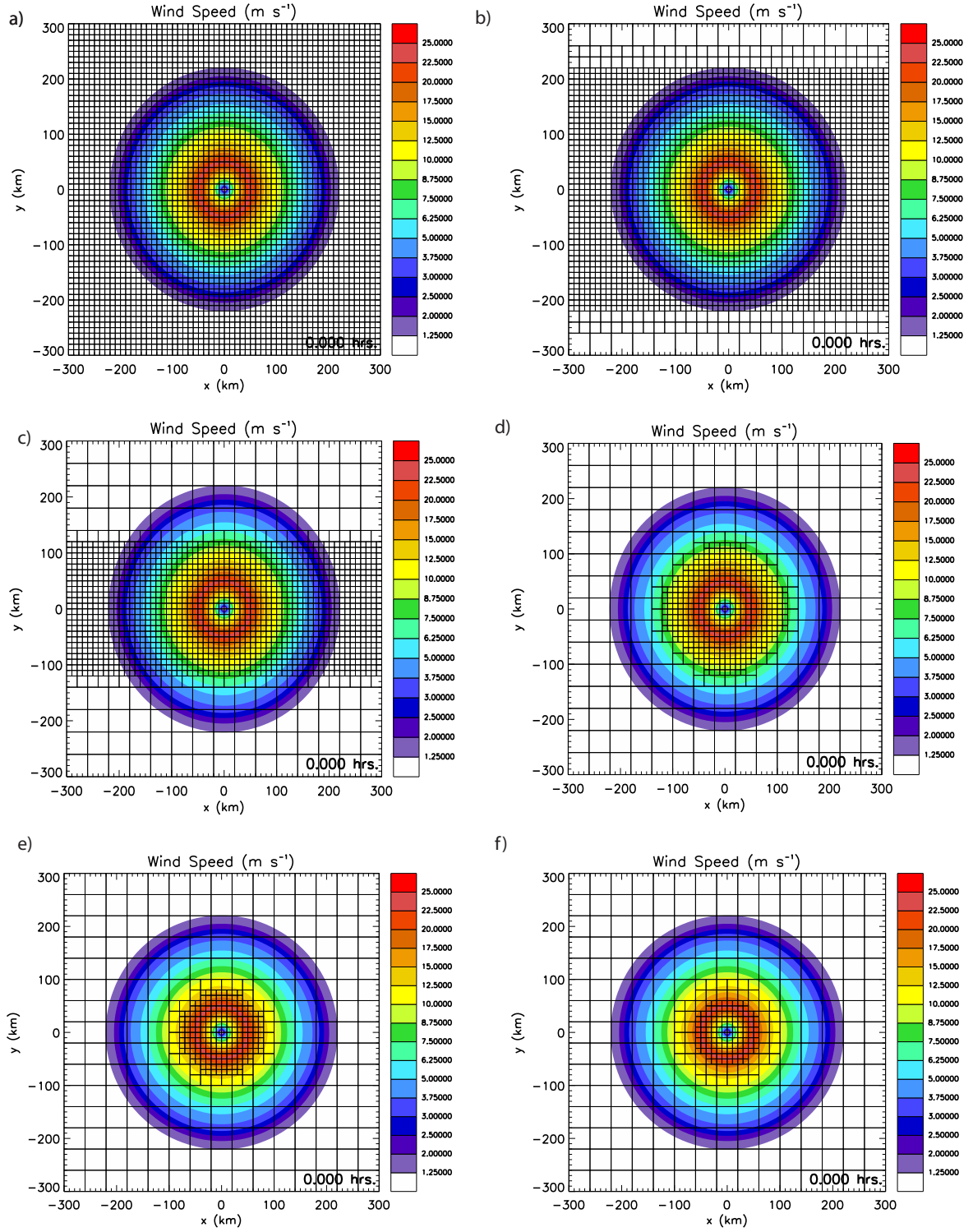


FIG. 3. Initial condition for case 1: advecting vortex. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.



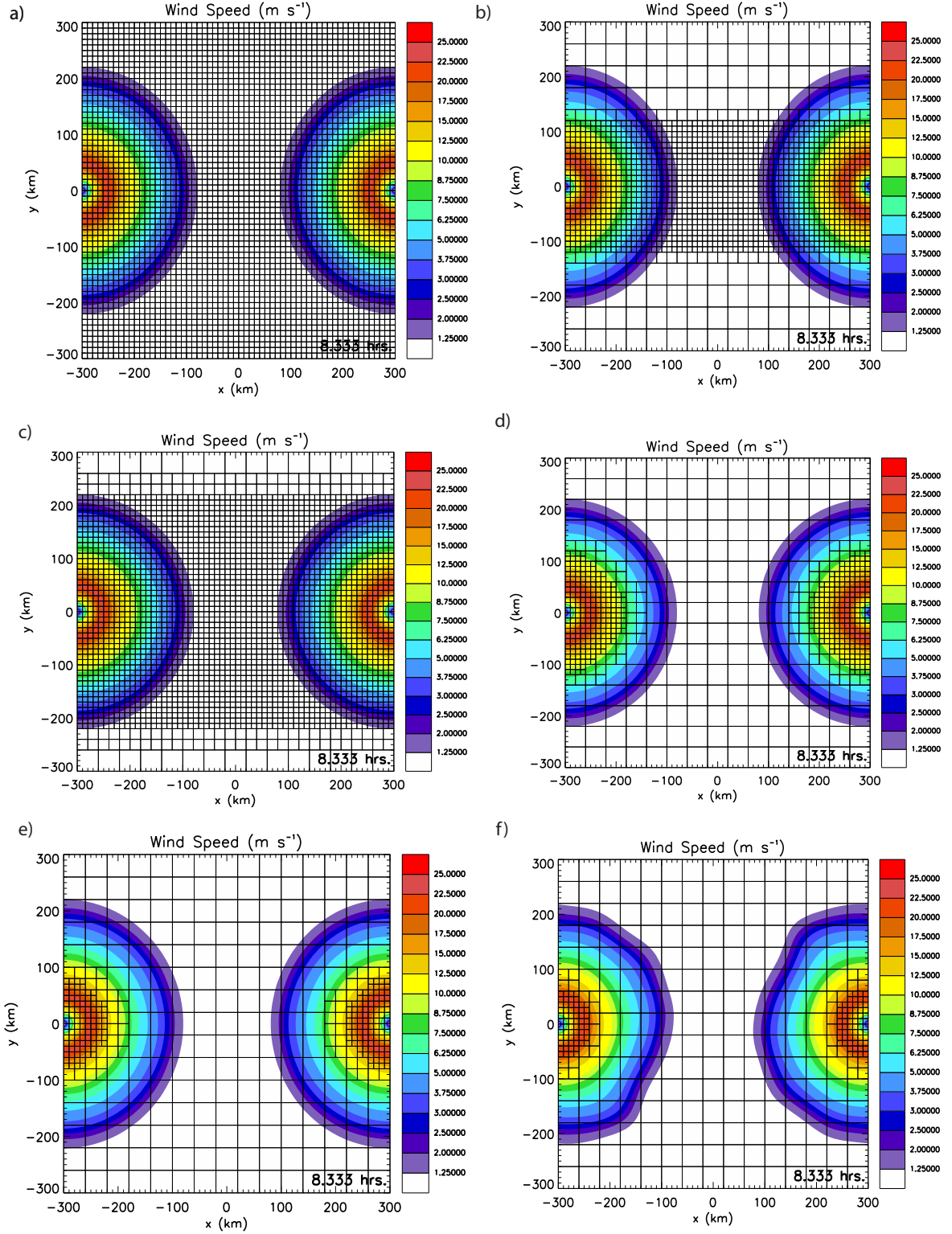


FIG. 4. Simulations of case 1 after one-half revolution, at  $t = 8.33$  h. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

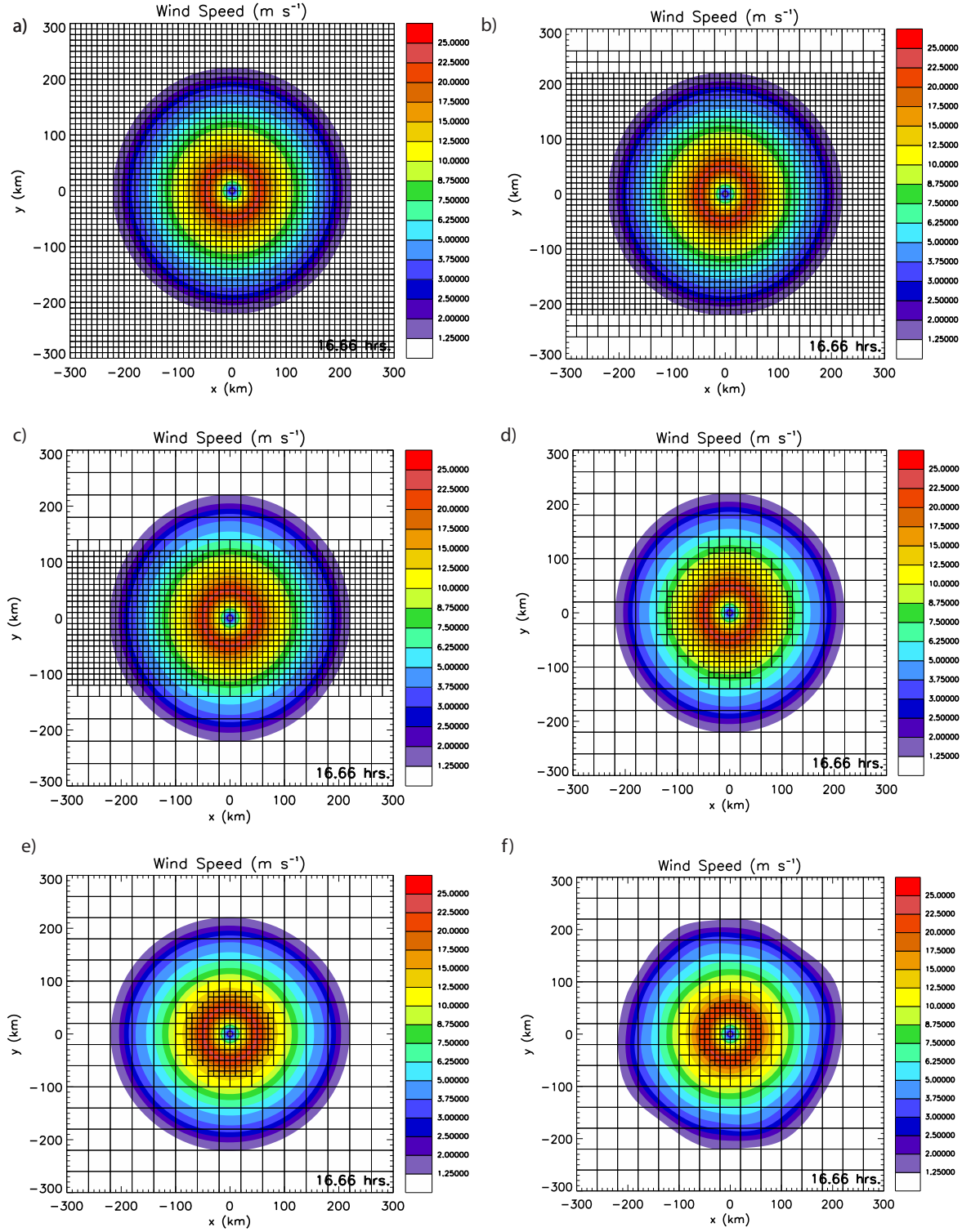


FIG. 5. Simulations of case 1 after a full revolution, at  $t = 16.67$  h. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.



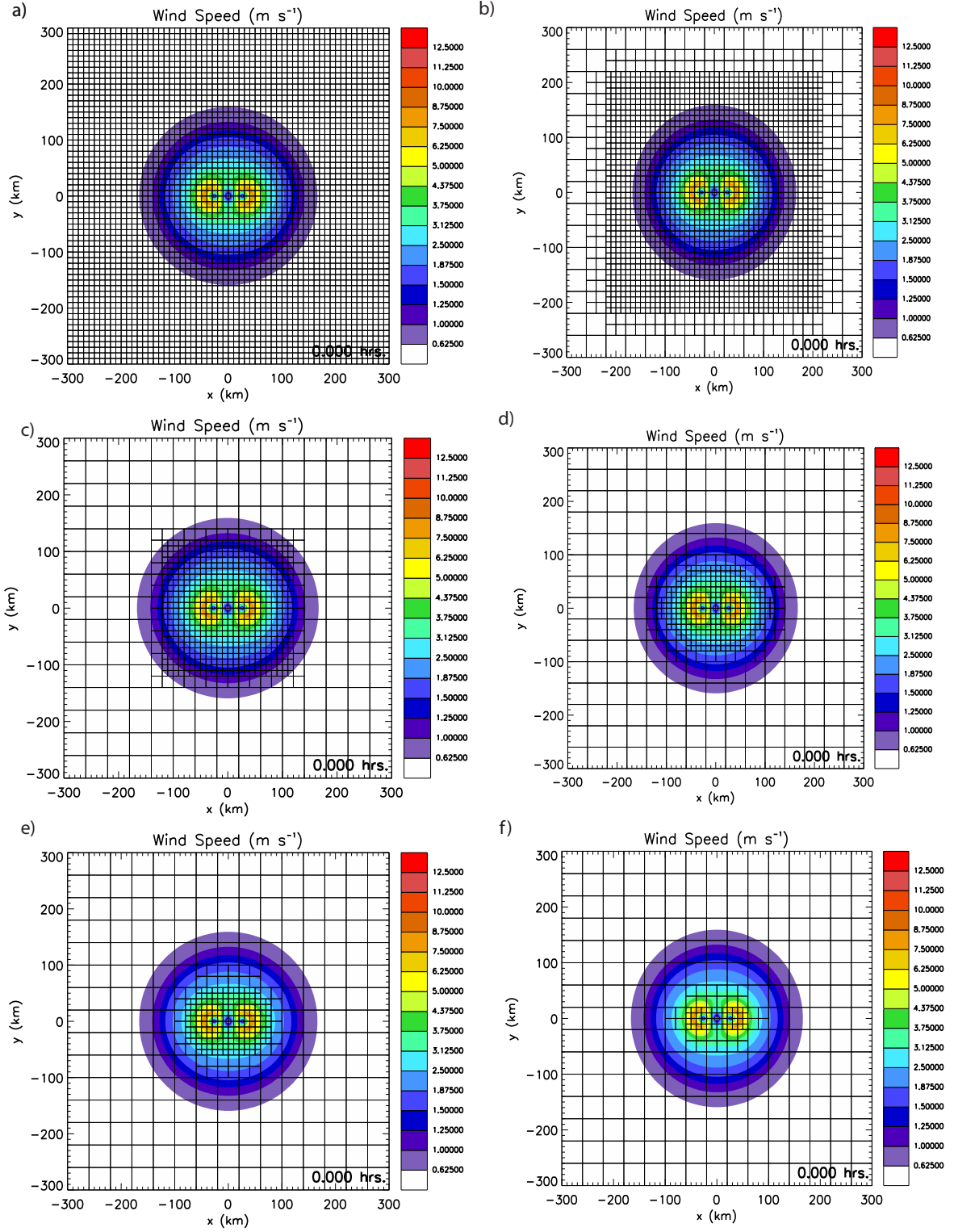


FIG. 6. Initial condition for case 2: Binary vortex interaction. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

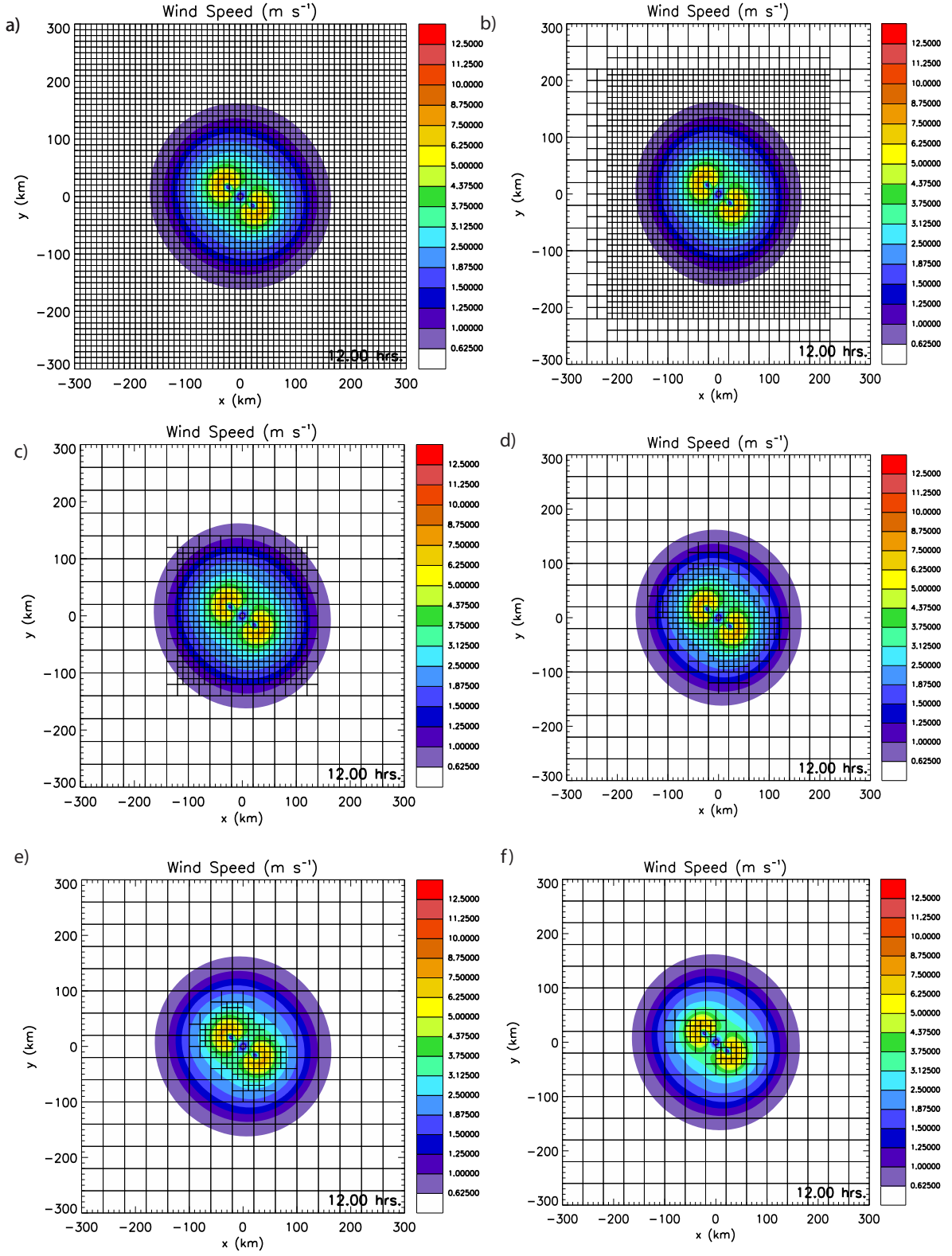


FIG. 7. Simulations of case 2 at  $t = 12$  h. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

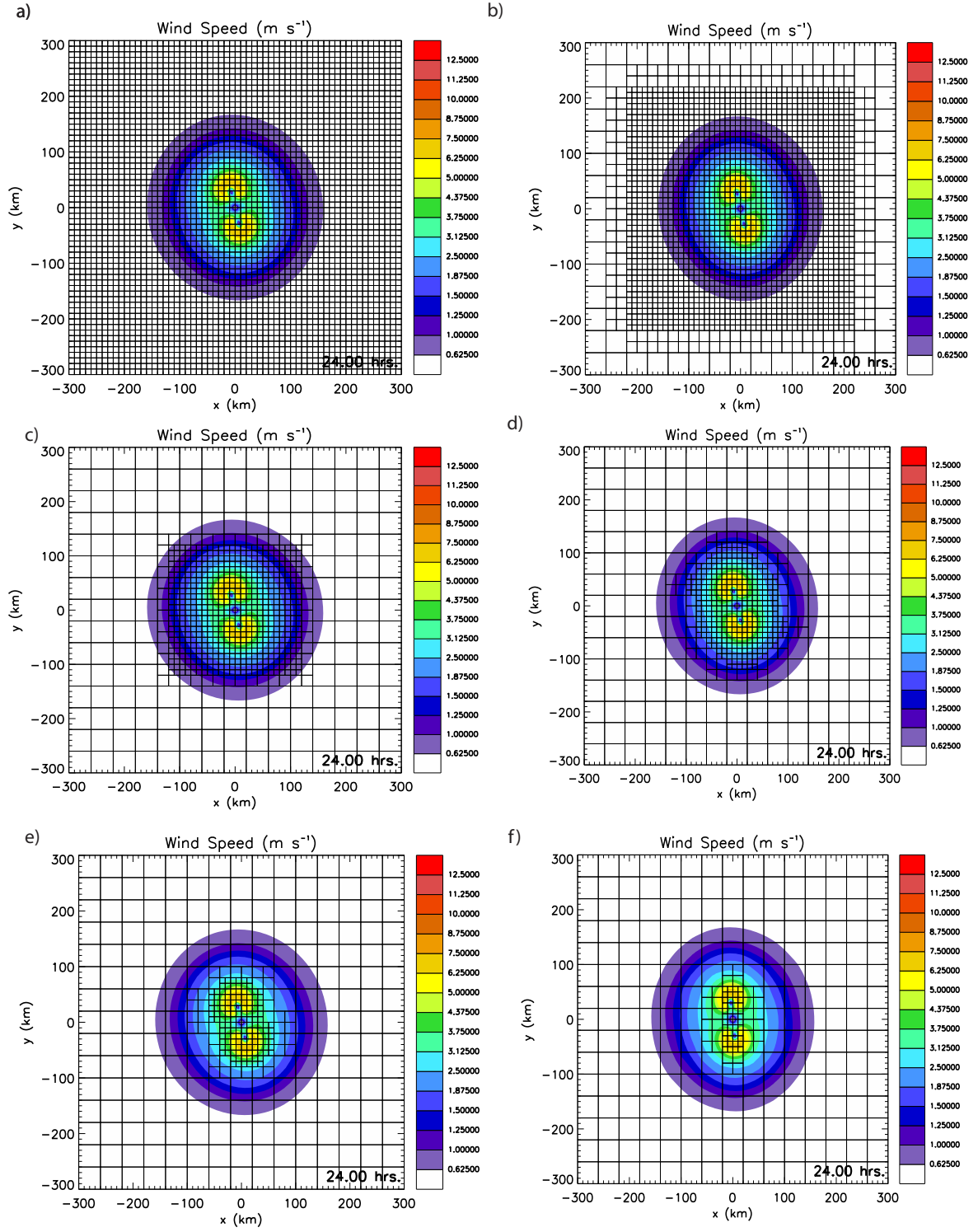


FIG. 8. Simulations of case 2 at  $t = 24$  h. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

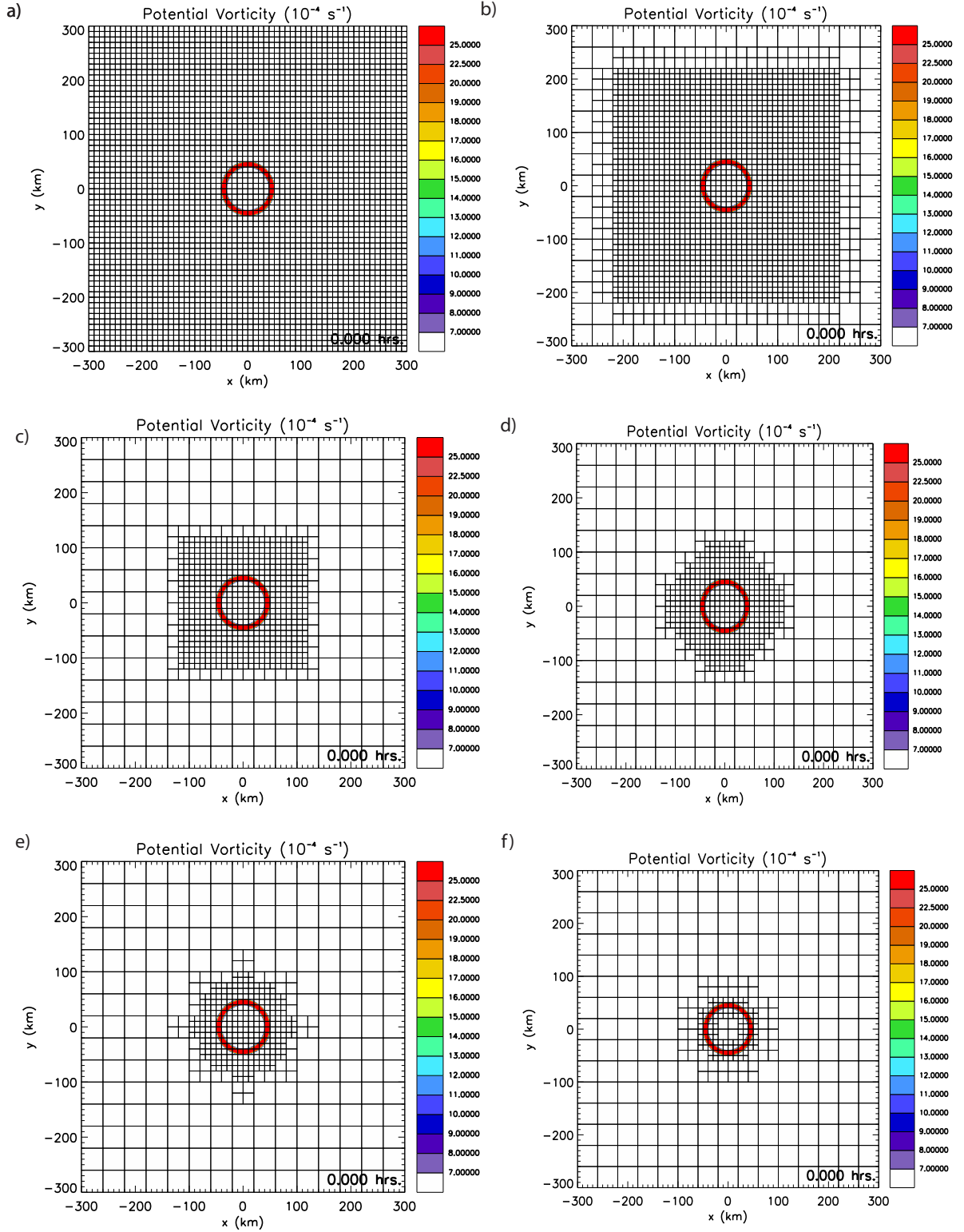


FIG. 9. Initial condition for case 3: unstable vortex. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

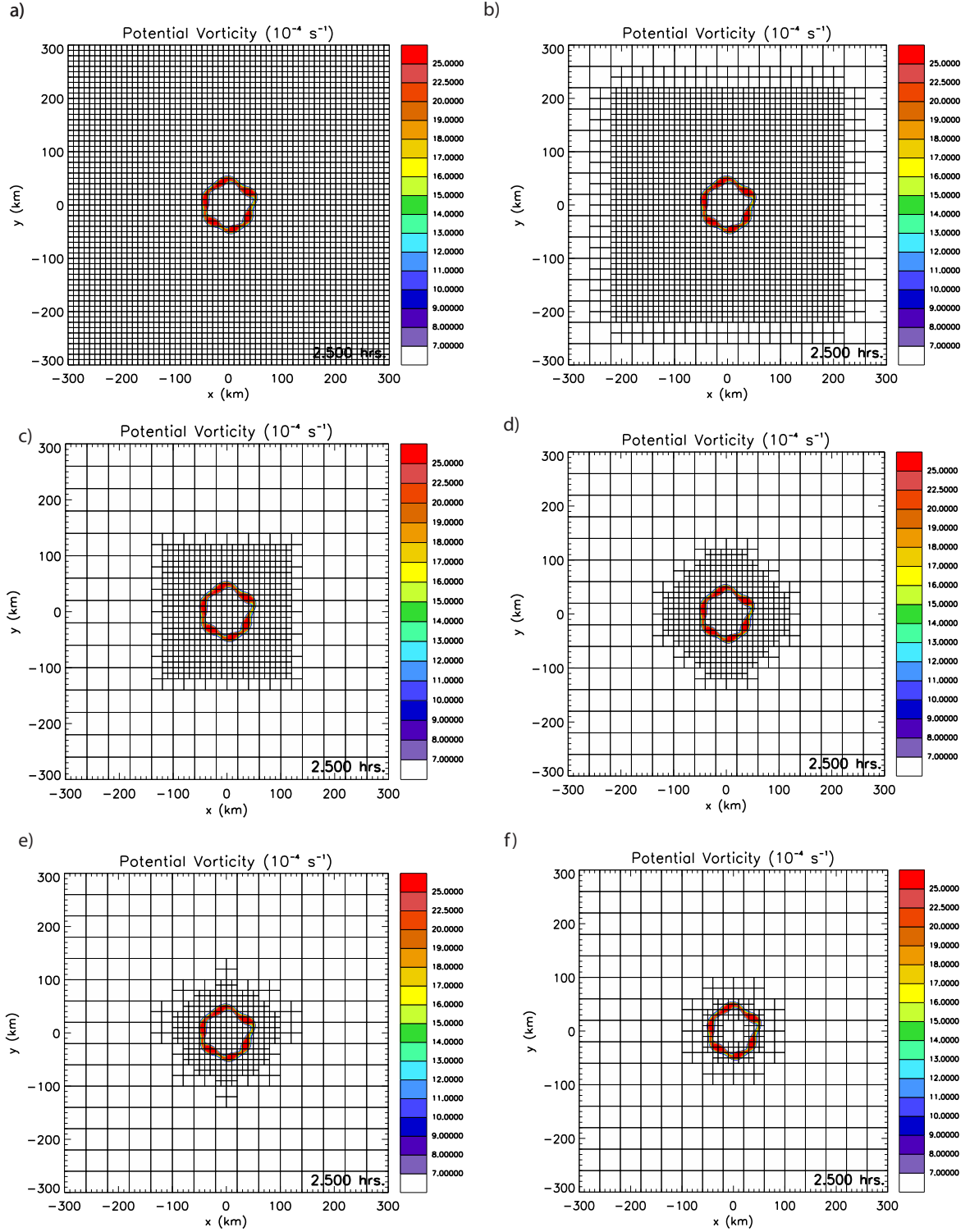


FIG. 10. Simulations of case 3 at  $t = 2.5 \text{ h}$  as barotropic instability occurs. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

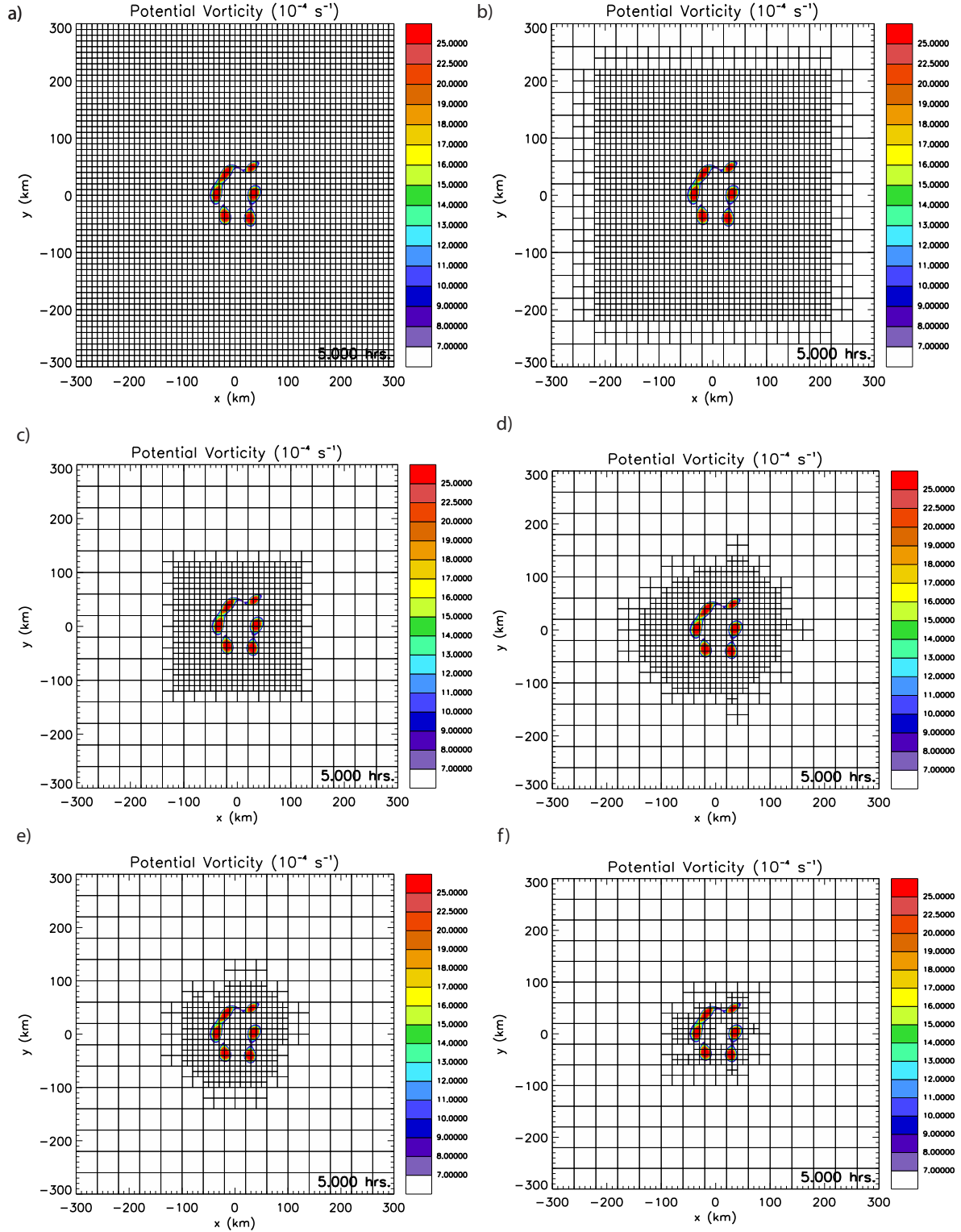


FIG. 11. Simulations of case 3 at  $t = 5 \text{ h}$  as mesovortices form. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

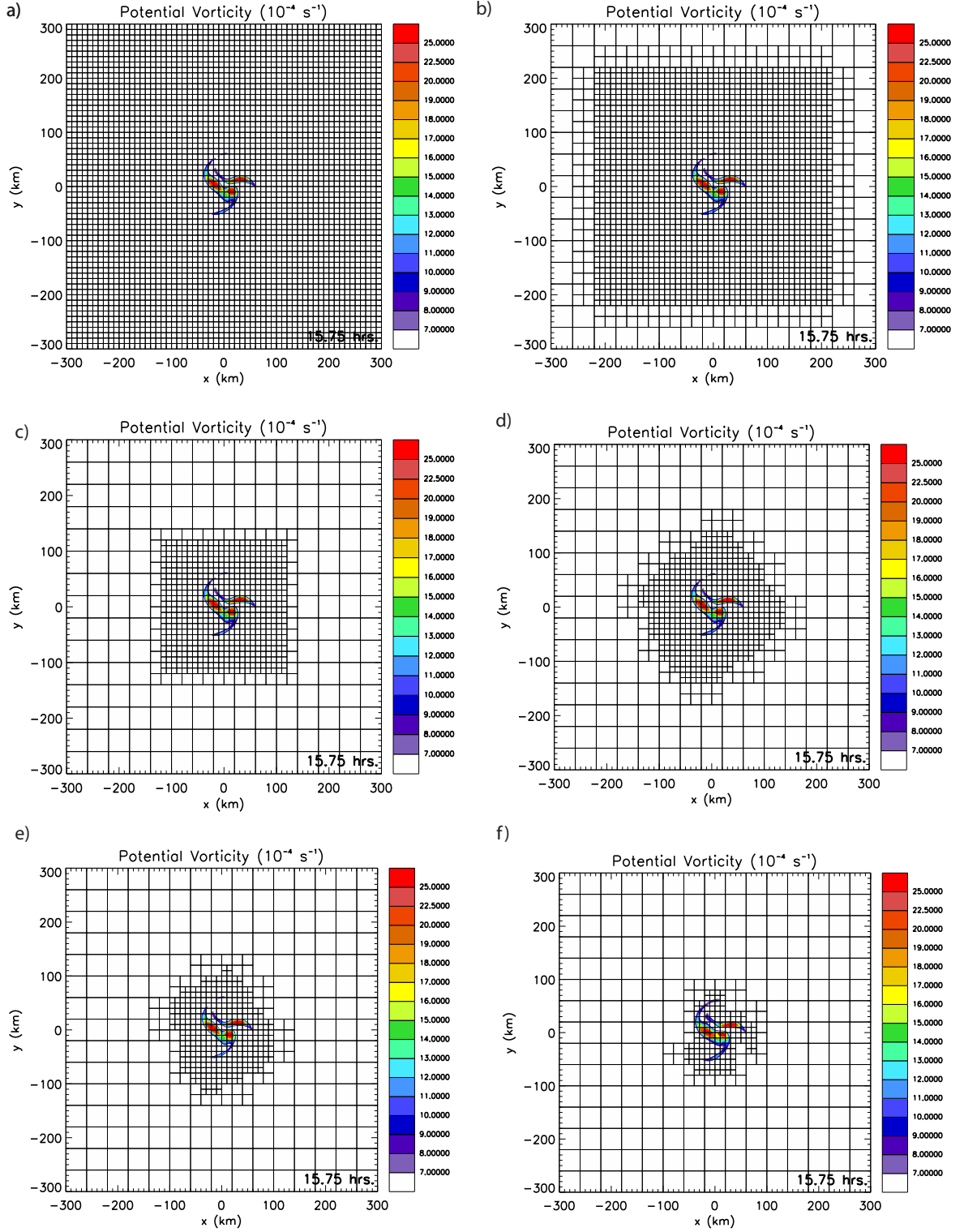


FIG. 12. Simulations of case 3 at  $t = 15.75 \text{ h}$  as the mesovortices merge into a monopole. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.



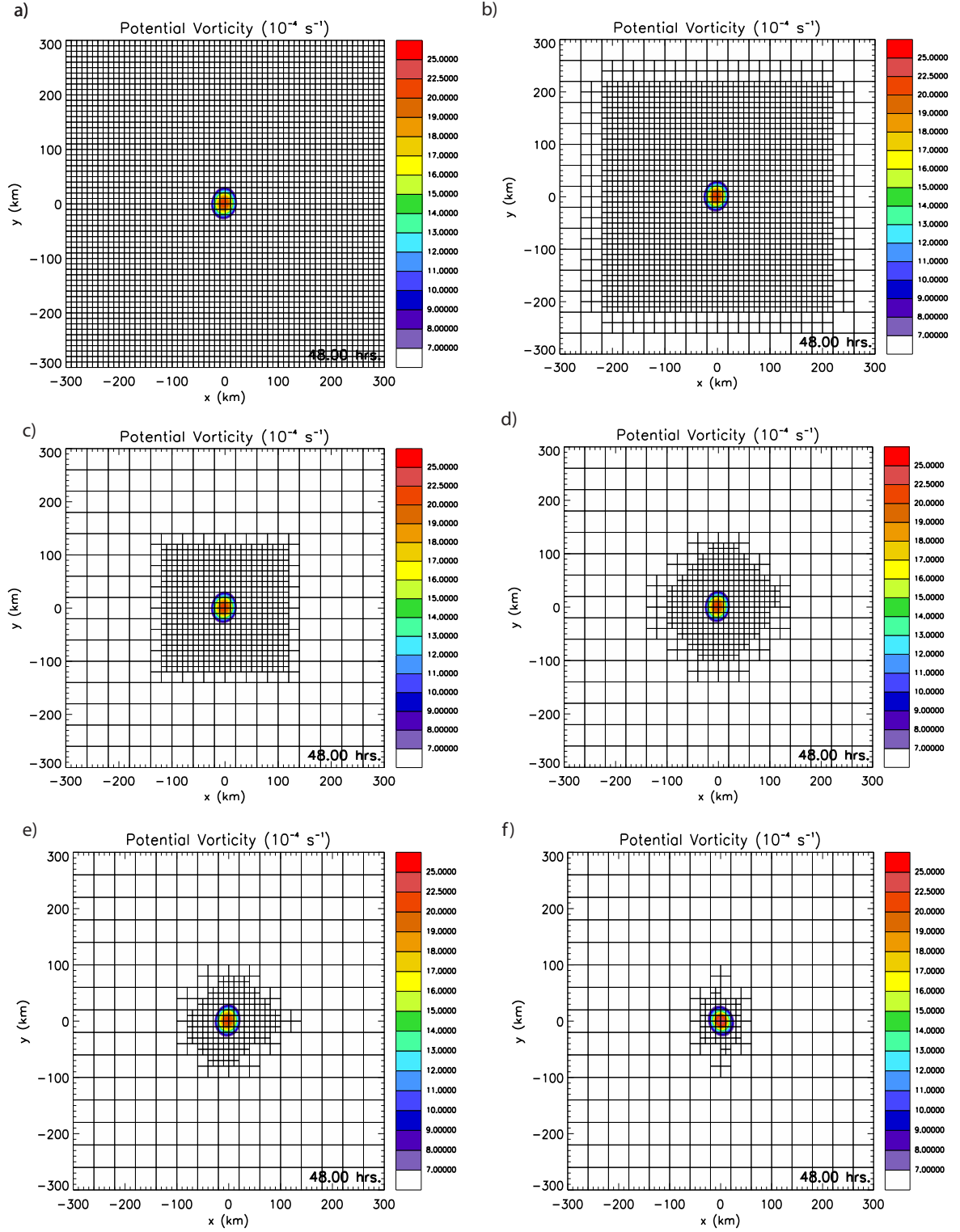


FIG. 13. Simulations of case 3 at  $t = 48 \text{ h}$  as the PV ring has broken down and mixed to a monopole. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.



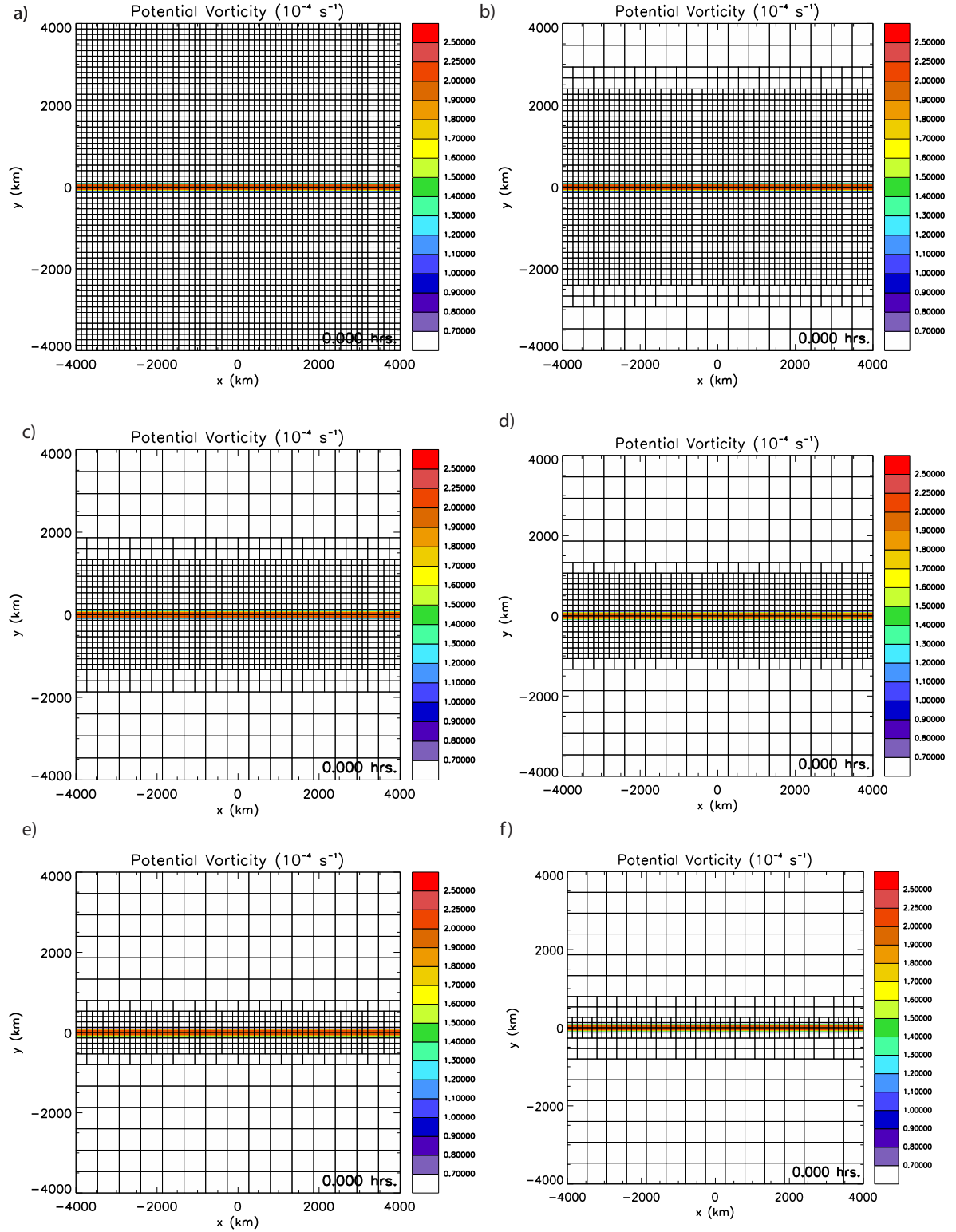


FIG. 14. Initial condition for case 4: strip of PV representing the ITCZ. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

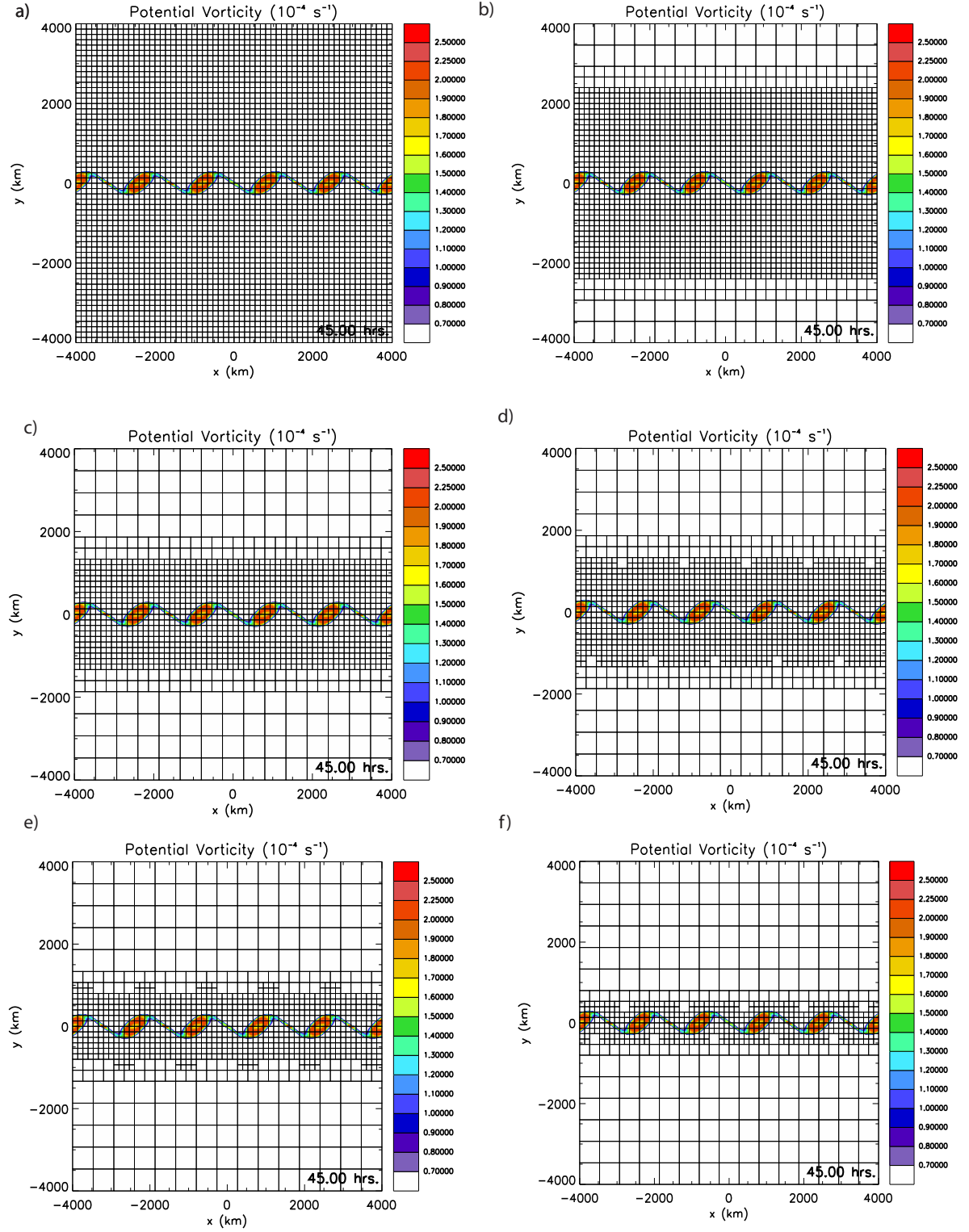


FIG. 15. Simulations of case 4 as barotropic instability sets in at  $t = 45$  h. Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

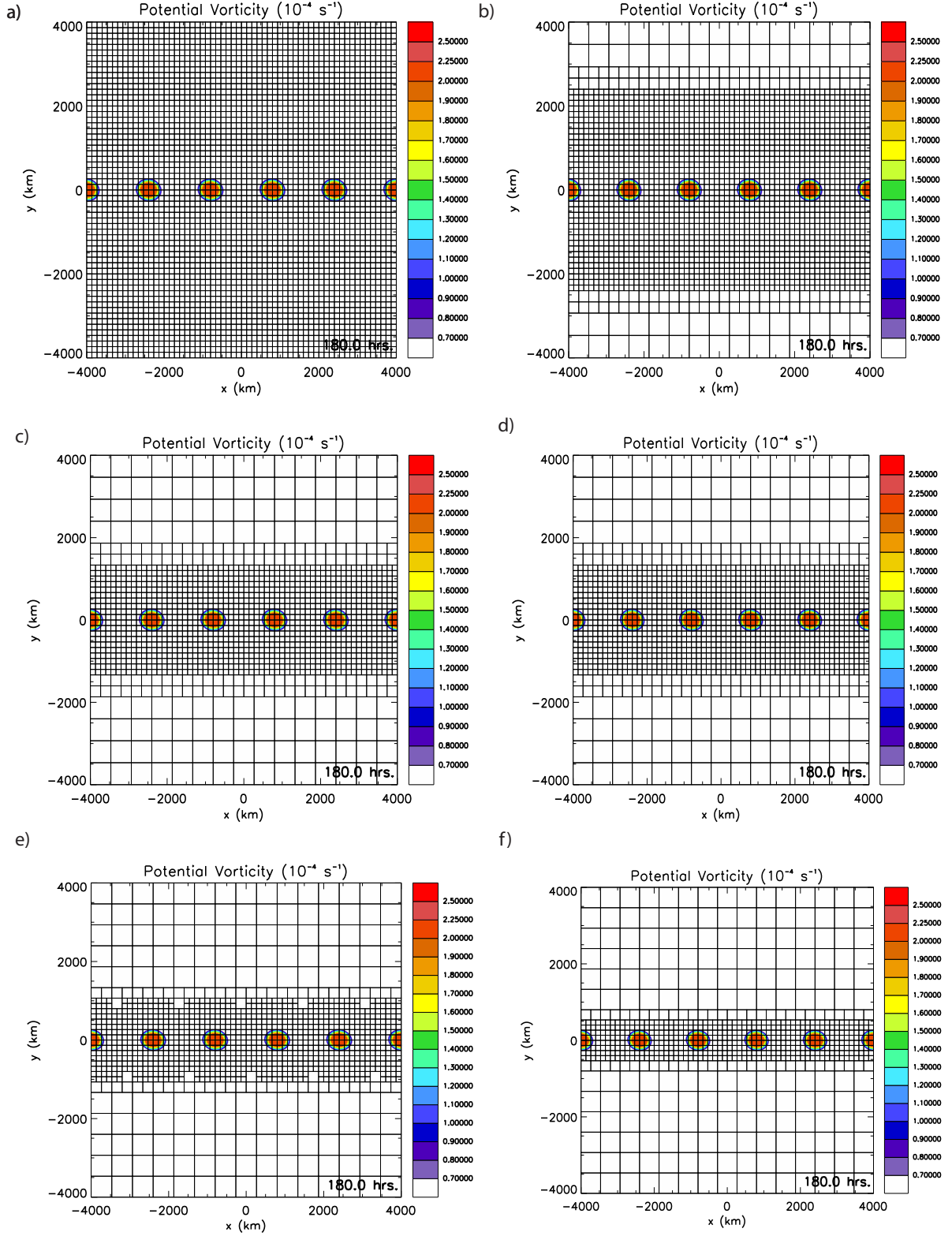


FIG. 16. Simulations of case 4 after multiple TC-like vortices have formed, at  $t = 180 \text{ h}$ . Panels: a) FINE, b) SMR2, c) SMR1, d) AMR3, e) AMR2, f) AMR1.

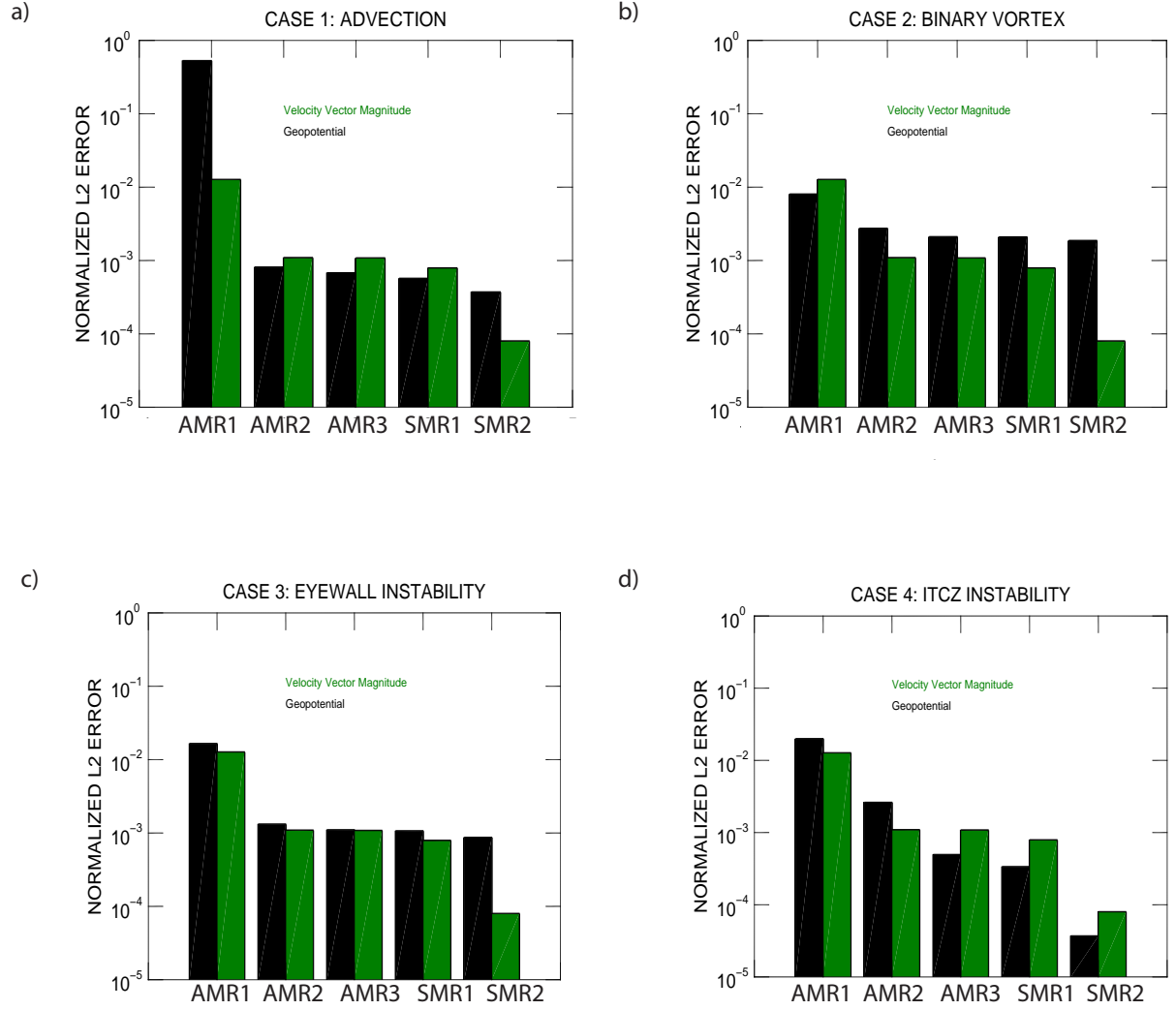


FIG. 17. Normalized L2 Errors for all cases for using all nodal points in the domain. Panels: a) case 1: advection, b) case 2: binary vortex, c) case 3: instability, d) case 4: ITCZ. The green bars represent the magnitude of the velocity vector and the black bars represent the geopotential.

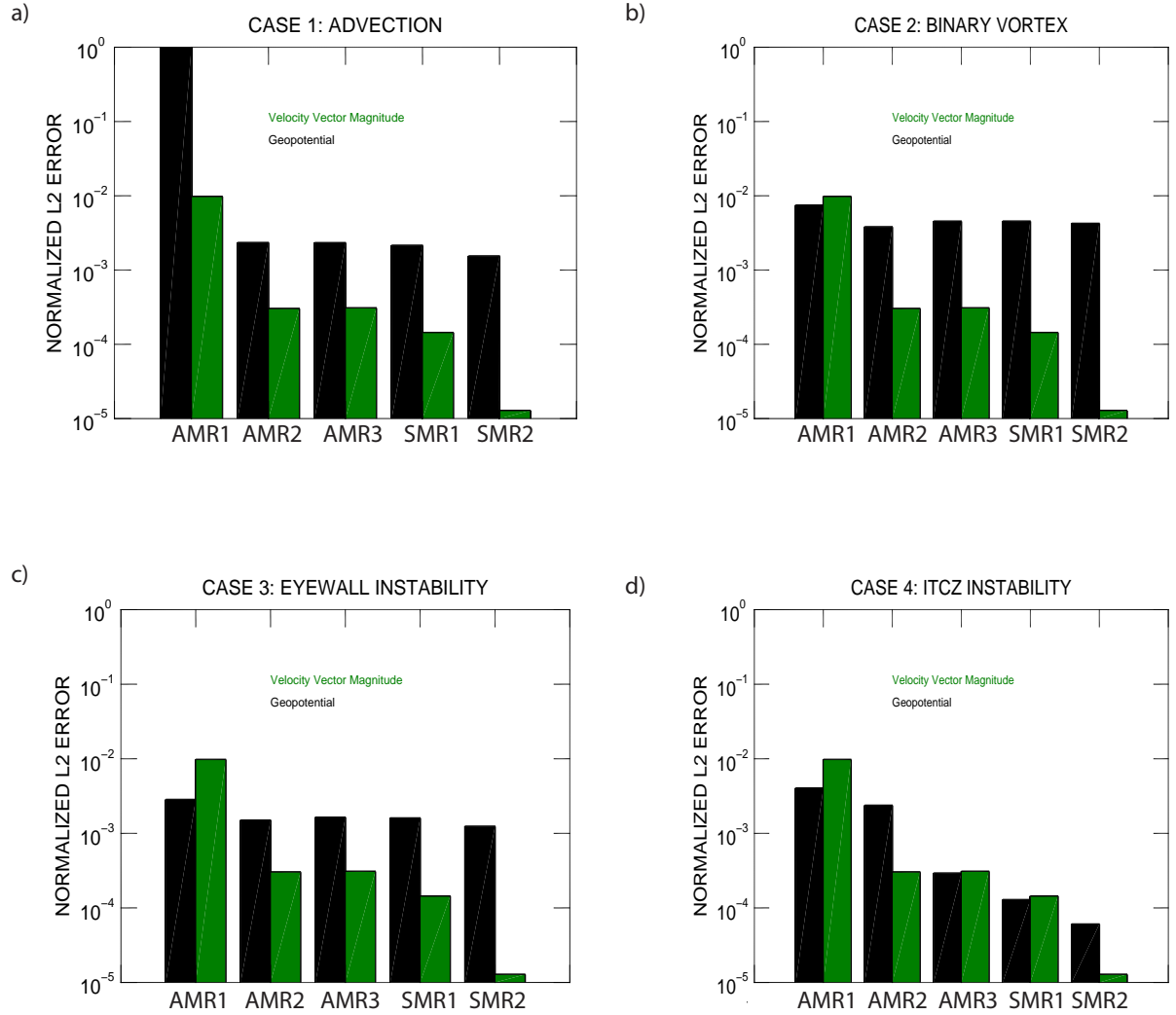
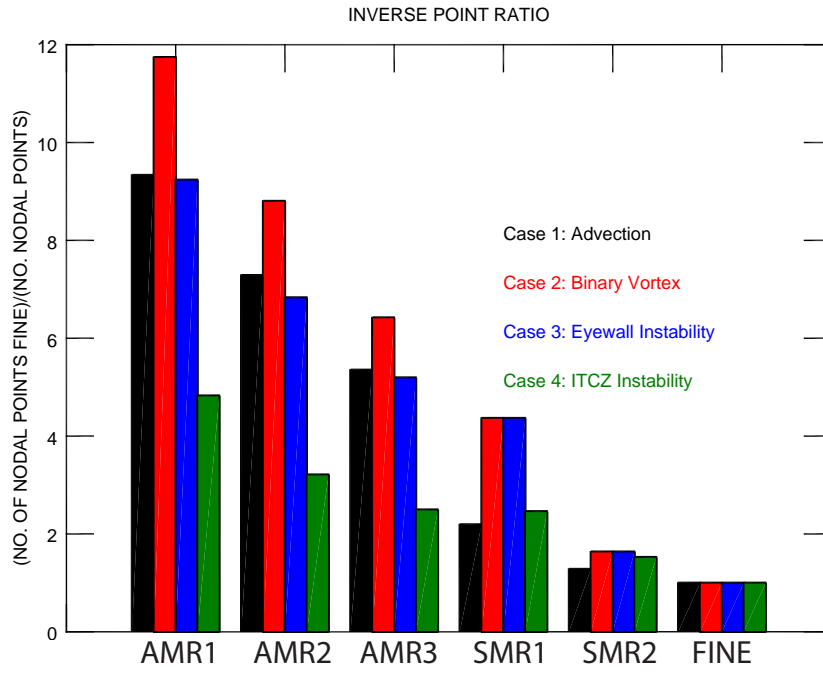


FIG. 18. Normalized L2 Errors for all cases using only nodal points in the localized region of high PV ( $r < 100$  km for cases 1–3,  $|y| < 500$  km for case 4). Panels: a) case 1: advection, b) case 2: binary vortex, c) case 3: instability, d) case 4: ITCZ. The green bars represent the magnitude of the velocity vector and the black bars represent the geopotential.

a)



b)

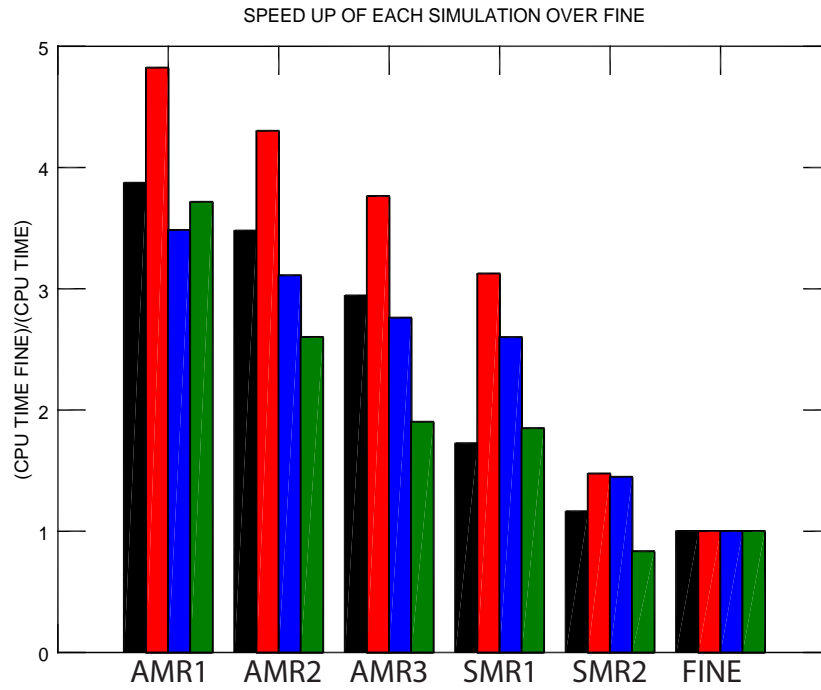


FIG. 19. Panels: a) Ratio of the number of FINE nodal points to the average number of nodal points in each of the other simulations, and b) Ratio of the CPU time of the FINE simulation to the CPU time of each of the other simulations.

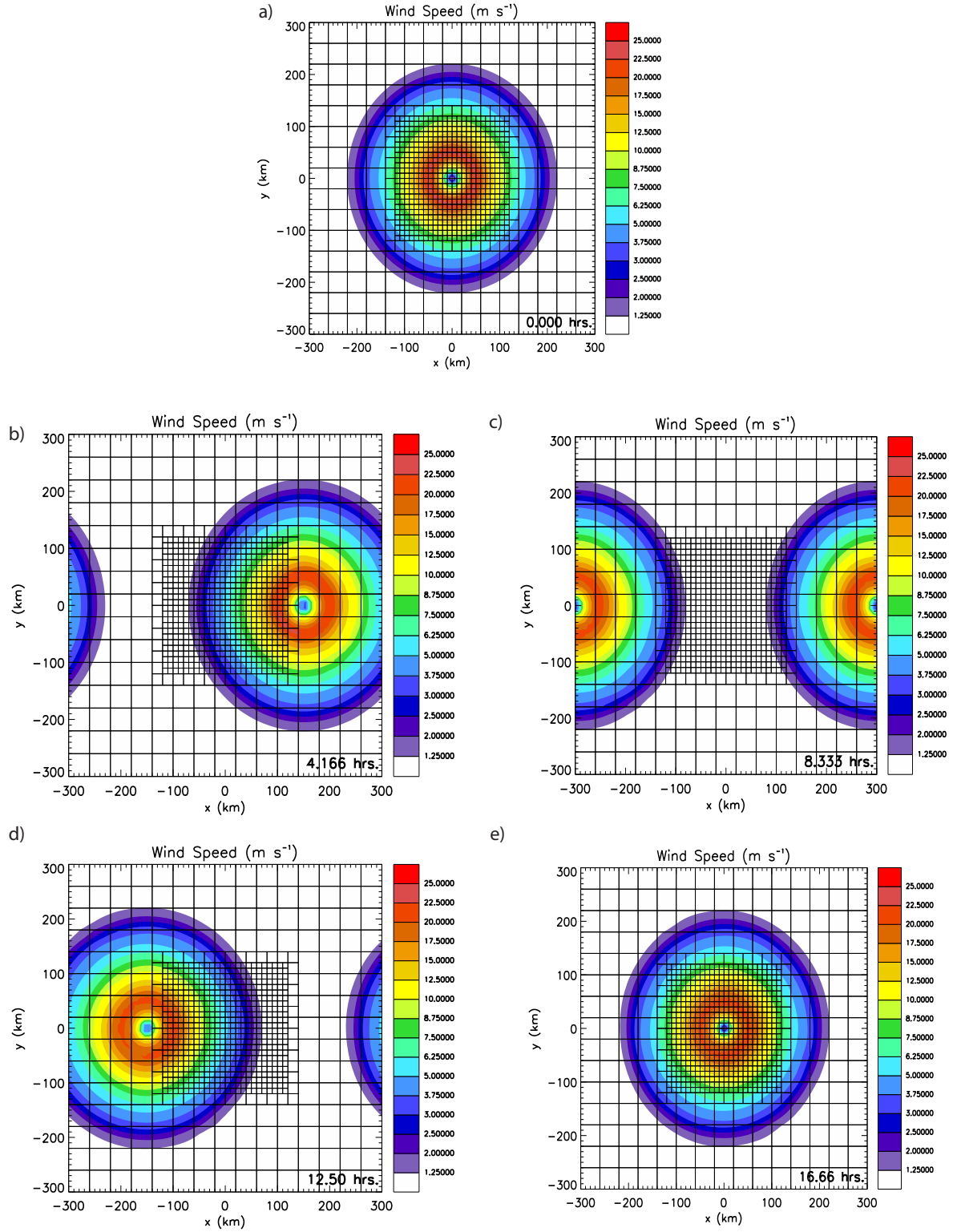


FIG. 20. TC vortex moving through a fixed variable mesh. Panels: a)  $t = 0$  h, b)  $t = 4.165$  h, c)  $t = 8.333$  h, d)  $t = 12.5$  h, e)  $t = 16.667$  h.